MA2E01 Tutorial problems #3
(due at the end of your tutorial)

1. Use the chain rule to compute the partial derivatives z_s, z_t in the case that

$$z = \ln(x^2 + y^3), \qquad x = s^2 t, \qquad y = \sin(st).$$

2. Consider the function $f(x, y, z) = x^2 e^{y/z}$ at the point $(3, 0, 1)$.

(a) What is the rate at which f is changing in the direction of $\boldsymbol{u} = \langle 1, 2, 1 \rangle$?

(b) Find a unit vector in the direction in which f increases most rapidly.

3. Consider the function $f(x, y) = \sqrt{\sin(x^2 y) + x^3 + 2y}$ at the point $(1, 0)$.

(a) Find the direction in which f is decreasing most rapidly.

(b) Find the equation of the tangent plane at the given point.

4. Suppose that $z = f(x - y, y - x)$ for some function f. Show that $z_x + z_y = 0$.