Lecture 3, September 28

- **Lines.** The line that passes through $A(a_1, a_2, a_3)$ with direction $\boldsymbol{v} = \langle v_1, v_2, v_3 \rangle$ can be described using the parametric equations

$$x = a_1 + tv_1, \qquad y = a_2 + tv_2, \qquad z = a_3 + tv_3.$$

- **Vector-valued functions.** A vector-valued function is one that has the form

$$\boldsymbol{r}(t) = \langle f(t), g(t), h(t) \rangle .$$

Its limits, derivatives and integrals may all be computed component-wise. Its graph is a curve in $\mathbb{R}^3$ and its derivative $\boldsymbol{r}'(t)$ is tangent to the curve at each point. If $\boldsymbol{r}(t)$ is the position of a moving object, then $\boldsymbol{r}'(t)$ is its velocity and $||\boldsymbol{r}'(t)||$ is its speed.

- **Tangent line.** The tangent line to the curve $\boldsymbol{r}(t)$ at time $t = t_0$ passes through the point $\boldsymbol{r}(t_0)$ with direction $\boldsymbol{r}'(t_0)$. One may determine its equation as above.

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Example 1. Consider the points $A(1, 2, 4)$ and $B(5, 3, 2)$. To determine the line that passes through these points, we note that the direction of this line is

$$\overrightarrow{AB} = \langle 5 - 1, 3 - 2, 2 - 4 \rangle = \langle 4, 1, -2 \rangle .$$

Since the line passes through $A(1, 2, 4)$ with that direction, its equation is then

$$x = 1 + 4t, \qquad y = 2 + t, \qquad z = 4 - 2t.$$

Example 2. Suppose that the position of a moving object is $\boldsymbol{r}(t) = \langle t^3, 2t^2, 5t \rangle$. Then its velocity vector is $\boldsymbol{r}'(t) = \langle 3t^2, 4t, 5 \rangle$. To find the tangent line when $t = 1$, we note that it passes through $(1, 2, 5)$ with direction $\boldsymbol{r}'(1) = \langle 3, 4, 5 \rangle$, so its equation is

$$x = 1 + 3t, \qquad y = 2 + 4t, \qquad z = 5 + 5t.$$

To compute the object's speed at time $t = 1$, we note that

$$\boldsymbol{r}'(1) = \langle 3, 4, 5 \rangle \quad \Longrightarrow \quad ||\boldsymbol{r}'(1)|| = \sqrt{3^2 + 4^2 + 5^2} = \sqrt{50} = 5\sqrt{2}.$$