Lecture 2, September 26

- **Vectors.** The vector that points from $A(a_1, a_2, a_3)$ to $B(b_1, b_2, b_3)$ is given by

$$\overrightarrow{AB} = \langle b_1 - a_1, b_2 - a_2, b_3 - a_3 \rangle .$$

Vector addition and scalar multiplication are defined component-wise:

$$\overrightarrow{v} + \overrightarrow{w} = \langle v_1 + w_1, v_2 + w_2, v_3 + w_3 \rangle , \qquad \lambda \overrightarrow{v} = \langle \lambda v_1, \lambda v_2, \lambda v_3 \rangle$$

whenever $\overrightarrow{v} = \langle v_1, v_2, v_3 \rangle$, $\overrightarrow{w} = \langle w_1, w_2, w_3 \rangle$ and λ is a scalar. Also, the expression

$$||\overrightarrow{v}|| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

gives the length (or norm) of $\overrightarrow{v}$, while similar formulas hold for vectors in $\mathbb{R}^2$.

- **Dot product.** The dot product of two vectors may be computed in two ways:

$$\overrightarrow{v} \cdot \overrightarrow{w} = v_1 w_1 + v_2 w_2 + v_3 w_3, \qquad \overrightarrow{v} \cdot \overrightarrow{w} = ||\overrightarrow{v}|| \cdot ||\overrightarrow{w}|| \cdot \cos\theta.$$

Here, the two vectors have the same starting point and θ is the angle between them.

- **Parallel/orthogonal.** Two vectors are parallel if and only if they are scalar multiples of one another. Two vectors are orthogonal if and only if their dot product is zero.

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Example 1. Consider the triangle with vertices $A(1, 0, 2)$, $B(5, 3, 4)$ and $C(3, -4, 4)$. To see that the angle at the point A is a right angle, we note that

$$\overrightarrow{AB} = \langle 4, 3, 2 \rangle , \qquad \overrightarrow{AC} = \langle 2, -4, 2 \rangle \quad \Longrightarrow \quad \overrightarrow{AB} \cdot \overrightarrow{AC} = 8 - 12 + 4 = 0.$$

To determine the angle at the point B, we use the formula

$$\cos B = \frac{\overrightarrow{BA} \cdot \overrightarrow{BC}}{||\overrightarrow{BA}|| \cdot ||\overrightarrow{BC}||} = \frac{\langle -4, -3, -2 \rangle \cdot \langle -2, -7, 0 \rangle}{\sqrt{4^2 + 3^2 + 2^2}\sqrt{2^2 + 7^2}} = \frac{29}{\sqrt{29}\sqrt{53}} = \sqrt{\frac{29}{53}}.$$

Example 2. Consider the points $A(1, 2, 3)$, $B(4, 2, 1)$ and $C(1, 1, 1)$. The vectors

$$\overrightarrow{AB} = \langle 3, 0, -2 \rangle , \qquad \overrightarrow{BC} = \langle -3, -1, 0 \rangle$$

are not parallel, as they are not scalar multiples of one another. This means that they describe different directions, so the three given points are not collinear.