Lecture 1, September 24

- **Area between two graphs.** If the graph of f lies above the graph of g, then the area that lies between the two graphs from $x = a$ to $x = b$ is given by

$$\text{Area} = \int_a^b [f(x) - g(x)]\, dx.$$

- **Volumes by slicing.** A solid is placed along the x-axis between $x = a$ and $x = b$. If its cross section at each point x has area $A(x)$, then the volume of the solid is

$$\text{Volume} = \int_a^b A(x)\, dx.$$

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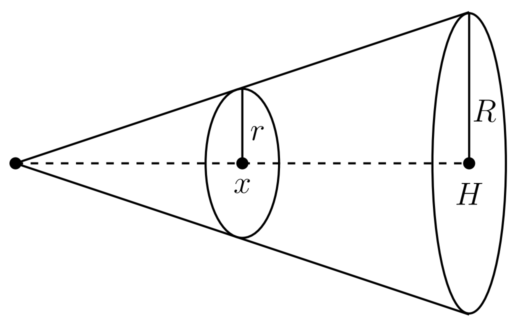
Example 1. Let $f(x) = 4x$ and $g(x) = x^2$. To compute the area of the region that lies between the two graphs, we note that the graphs intersect when

$$4x = x^2 \quad \Longrightarrow \quad x(x-4) = 0 \quad \Longrightarrow \quad x = 0, 4.$$

Since a quick sketch shows that the graph of f lies above the graph of g between the two points of intersection, the area of the desired region is

$$\text{Area} = \int_0^4 (4x - x^2)\, dx = \left[2x^2 - \frac{x^3}{3}\right]_0^4 = 32 - \frac{64}{3} = \frac{32}{3}.$$

Example 2. Consider a cone of radius R and height H. To place such a cone along the x-axis, we put its vertex at the origin and the center of its base at $(H, 0)$.



The cross section of the cone at each point x is then a circle of radius r, where

$$\frac{r}{x} = \frac{R}{H} \quad \Longrightarrow \quad r = \frac{Rx}{H}$$

by similar triangles. Since the cross section has area $\pi r^2 = \pi R^2 x^2 / H^2$, we get

$$\text{Volume} = \int_0^H \frac{\pi R^2 x^2}{H^2}\, dx = \frac{\pi R^2 H^3}{3H^2} = \frac{\pi R^2 H}{3}.$$