Lecture 5, October 3

• Functions of two variables. The domain of a function z = f(x, y) is the set of all points (x, y) at which it is defined. The graph of such a function is a surface in \mathbb{R}^3 . To draw a rough sketch of the graph, one looks at the level curves f(x, y) = k for various values of k. These are curves in the xy-plane that correspond to horizontal slices of the graph; they describe the part of the graph which lies at height z = k.

.....

Example 1. Consider the function $z = x^2 + y^2$. Then the level curve z = 1 is

$$z = 1 \implies x^2 + y^2 = 1,$$

the circle of radius 1 around the origin. One can easily draw this in the xy-plane, as we do below in the left part of the figure. Similarly, the level curve z = 4 is

$$z = 4 \implies x^2 + y^2 = 4,$$

the circle of radius 2 around the origin. To get a rough sketch of the graph, recall that the first circle is the horizontal slice at z = 1, while the second circle is the horizontal slice at z = 4. Imagine lifting the first one up by 1 unit and the second one up by 4 units. When lifted, the circles around the origin become circles around the z-axis and the overall shape of the graph is the one depicted below.



Figure 1: The level curves $x^2 + y^2 = k$ and the graph of $z = x^2 + y^2$.