MA2327, Homework #2

due Thursday, Oct. 27 or Nov. 3

1. Let $x_0, y_0 \in \mathbb{R}$ be given and consider the initial value problem

$$y'(x) = \frac{xy(x)}{x-1}, \qquad y(x_0) = y_0.$$

For which values of x_0, y_0 is the solution unique? What can you say for the remaining values? Hint: A similar problem appears in the notes online; see Chapter 1, page 30.

2. Let $a > 0$ be given and consider the initial value problem

$$y'(x) = \frac{x^2+3}{x^3+x} \cdot y(x) \cdot \sin y(x), \qquad y(1) = a.$$

Use the associated integral equation to show that $|y(x)| \leq \frac{2ax^3}{x^2+1}$ for all $x \geq 1$. Hint: the sine term is at most 1; use the Gronwall inequality and then partial fractions.

3. Find a basis of solutions for the linear homogeneous system

$$\boldsymbol{y}'(t) = A(t)\boldsymbol{y}(t), \qquad A(t) = \begin{bmatrix} 2t/(t^2+1) & 0 \\ 2t & 2t \end{bmatrix}.$$

4. Use the eigenvector method to solve the linear homogeneous system

$$\left\{ \begin{array}{l} x'(t) = 5x(t) + 2y(t) - 4z(t) \\ y'(t) = 4x(t) + 7y(t) - 8z(t) \\ z'(t) = 4x(t) + 2y(t) - 3z(t) \end{array} \right\}.$$