Maths 212, Homework #8
First four problems:
due Thursday, Jan. 19

55. Show that the continuous image of a Hausdorff, limit point compact topological space is limit point compact itself.

56. Suppose that X is a limit point compact topological space and let $Y \subset X$ be closed in X. Show that Y is limit point compact itself.

57. Let X denote the complement of the unit circle in $\mathbb{R}^2$. Show that there is no path
$$\gamma\colon [0,1] \to X$$
between a point (x_0, y_0) in the interior of the circle and a point (x_1, y_1) in the exterior.

58. Given an arbitrary metric space X, it is clear that the collection of open balls
$$\mathcal{U} = \{B_1(x) \, : \, x \in X\}$$
forms an open cover of X. Find a Lebesgue number for this open cover.

59. A topological space X is called *countably compact* if every countable open cover of X has a finite subcover. Show that every sequentially compact space is countably compact.

60. Show that every countably compact topological space is limit point compact.

61. Find a countably compact metric space which is not compact.

62. Define a relation $\sim$ between points of a topological space X by the formula
$$x \sim y \iff x, y \text{ lie in some connected subset of } X.$$
Show that $\sim$ is an equivalence relation and that each equivalence class (component) of X is necessarily connected.