Maths 212, Homework #6
First four problems:
due Thursday, Dec. 1

38. Given a topological space X and a subset $A \subset X$, show that $X - \operatorname{Cl} A = \operatorname{Int}(X - A)$.

39. Suppose X is compact and Hausdorff. Show that a subset $A \subset X$ is compact if and only if it is closed in X.

40. Given a bounded subset A of $\mathbb{R}$, show that $\sup A$ is in the closure of A. Use this fact to conclude that every compact subset of $\mathbb{R}$ contains its supremum.

41. Let A be a compact subset of a Hausdorff space X and let $x \in X - A$. Show that there exist disjoint open sets U and V containing A and x, respectively.

42. Let A, B be disjoint compact subsets of a Hausdorff space X. Use the previous problem to show that there exist disjoint open sets U and V containing A and B, respectively.

43. Let $f\colon X \to Y$ be a continuous, bijective function from a compact space X to a Hausdorff space Y. Show that f is necessarily a homeomorphism.[1]

44. Let A, B be compact subsets of a Hausdorff space X. Show that their intersection $A \cap B$ is also compact.

45. Suppose X is compact and let $C_1, C_2, C_3, \ldots$ be nonempty closed subsets of X such that

$$C_1 \supset C_2 \supset C_3 \supset \cdots .$$

Show that the intersection of these sets is also nonempty.

[1] You need only check that f^{-1} is continuous. One way to do this would be to check that $f(U)$ is open in Y whenever U is open in X. Another way would be to check that $f(U)$ is closed in Y whenever U is closed in X. Needless to say, one way might be much easier than the other.