Maths 212, Homework #5
First four problems:
due Thursday, Nov. 24

30. Find the interior and closure of the following subsets of $\mathbb{R}^2$ (with the usual topology). You need not justify your answers.

    (a) $A = \{(x,y) \in \mathbb{R}^2 \,:\, x \in \mathbb{Q}, \quad y > 0\}$ (b) $B = \{(x,y) \in \mathbb{R}^2 \,:\, x^2 + y^2 \neq 4x\}$

31. Given a topological space X, show that $\operatorname{Cl} A \subset \operatorname{Cl} B$ whenever $A \subset B \subset X$.

32. Given a topological space X, show that $\operatorname{Bd} A$ is closed in X for each $A \subset X$.

33. Show that a finite subset of a metric space has no limit points. Use this fact to conclude that every finite subset of a metric space X is closed in X.

34. Find a subset A of $\mathbb{R}$ (with its usual topology) such that $\operatorname{Int}(\operatorname{Cl} A) \neq \operatorname{Cl}(\operatorname{Int} A)$.

35. Show that a function $f\colon X \to Y$ between topological spaces is continuous if and only if

$$f(\operatorname{Cl} A) \subset \operatorname{Cl} f(A) \qquad \text{for each } A \subset X.$$

As a hint, you might want to rewrite the last equation in the form

$$\operatorname{Cl} A \subset f^{-1}(\operatorname{Cl} f(A)) \qquad \text{for each } A \subset X.$$

36. Given a topological space X, show that $\operatorname{Int} A \cap \operatorname{Bd} A = \varnothing$ for each $A \subset X$.

37. Given a topological space X, show that $\operatorname{Int} A \cup \operatorname{Bd} A = \operatorname{Cl} A$ for each $A \subset X$.