Maths 212, Homework #4
First four problems:
due Thursday, Nov. 17

22. Define $\mathscr{B}$ as the collection of all intervals of the form $(-\infty, a)$ for some $a \in \mathbb{R}$.
    (a) Check that $\mathscr{B}$ is a basis for some topology on $\mathbb{R}$.
    (b) Compare this topology with the usual topology on $\mathbb{R}$. Which one is finer?
23. Let $f\colon X \to Y$ be an arbitrary function between topological spaces.
    (a) For which topology on X is f necessarily continuous? Explain.
    (b) For which topology on Y is f necessarily continuous? Explain.
24. Let X be a topological space and let $Y \subset X$. Given a basis $\mathscr{B}_X$ for the topology on X, show that $\mathscr{B}_Y = \{B \cap Y \,:\, B \in \mathscr{B}_X\}$ is a basis for the subspace topology[1] on Y.
25. Is the set $A = \{0 < x < 1 \,:\, 1/x \notin \mathbb{Z}\}$ open in $\mathbb{R}$ with the usual topology? Explain.
26. Show that a function $f\colon X \to Y$ is continuous if $f^{-1}(B) \in \mathscr{T}_X$ whenever $B \in \mathscr{B}_Y$. Here, $\mathscr{T}_X$ stands for the topology on X, while $\mathscr{B}_Y$ is any basis for the topology on Y.
27. Show that the product topology on $\mathbb{R}^2$ coincides with the usual topology on $\mathbb{R}^2$.
28. Given a metric space $\{X, d\}$, show that its metric $d\colon X \times X \to \mathbb{R}$ is continuous. Here, X is given the metric topology, $X \times X$ the product topology and $\mathbb{R}$ its usual topology.
29. Show that $X \times Y$ is homeomorphic to $Y \times X$ for all nonempty sets X, Y.

[1] Do note the difference between a basis for some unspecified topology and a basis for some specific topology. In the former case, it suffices to check properties (B1) and (B2). In the latter case, one has to check that every element of the given topology is a union of elements of the given basis.