Maths 212, Homework #3
First three problems:
due Thursday, Nov. 3

15. Decide which of the following sets U are open in X. You need not justify your answers.

    (i) the set $U = [0, 2)$ as a subset of $X = \mathbb{R}$ with the usual metric;

    (ii) the set $U = [0, 2)$ as a subset of $X = [0, 3]$ with the usual metric;

    (iii) the set $U = \{0, 2\}$ as a subset of $X = \mathbb{R}$ with the usual metric;

    (iv) the set $U = \{0, 2\}$ as a subset of $X = \mathbb{R}$ with the discrete metric.

16. Suppose $f : X \to Y$ and $g : Y \to Z$ are continuous functions between metric spaces. Show that their composition $g \circ f : X \to Z$ is also continuous.

17. Let $f : X \to Y$ be a constant function between metric spaces, say

$$f(x) = y_0 \qquad \text{for all } x \in X.$$

    Show that f is continuous. As a hint, take an open set U in Y and find its inverse image; you will need to consider two cases here.

18. Let X be a metric space. Show that a subset $U \subset X$ is open in X if and only if it is a union of open balls.

19. Show that the unit square

$$S = \{(x, y) \in \mathbb{R}^2 \;:\; 0 < x < 1, \quad 0 < y < 1\}$$

    is open in $\mathbb{R}^2$ with the usual metric.

20. (Restrictions) Suppose $f : X \to Y$ is a continuous function between metric spaces. Show that the restriction $f|_A : A \to Y$ to a subset of X is also continuous. Here, the restriction is defined by the formula

$$f|_A(x) = f(x) \qquad \text{for all } x \in A.$$

21. (Inclusions) Let X, Y be two metric spaces with $X \subset Y$. Show that the inclusion function $i : X \to Y$ is continuous. Here, the inclusion is defined by the formula

$$i(x) = x \qquad \text{for all } x \in X.$$