Maths 212, Homework #16
First three problems:
due Thursday, Apr. 13

117. Find the fundamental groups of the following spaces; you need not justify your answers.

(a) the unit disk B^2 with the origin removed;

(b) the complement of the z-axis in $\mathbb{R}^3$;

(c) the complement of the unit ball $B_1(0)$ in $\mathbb{R}^3$;

(d) the unit sphere S^2 with two points removed.

118. Suppose X is a retract of the unit disk B^2. Show that every continuous map $f\colon X \to X$ has a fixed point.

119. Let t be a fixed real number. Show that there exist real numbers x, y such that

$$5x = (x^2 + x)\cos t - y^2 \sin t,$$
$$5y = (x^2 - 1)\sin t + y^3 \cos t.$$

120. Compute the winding number of the loop $\gamma_0\colon [0,1] \to \mathbb{C} - \{0\}$ defined by $\gamma_0(t) = e^{4\pi i t}$.

121. Compute the winding number of the loop $\gamma_1\colon [0,1] \to \mathbb{C} - \{0\}$ defined by $\gamma_1(t) = 2e^{4\pi i t}$.

122. Show that path homotopic loops $\gamma_0, \gamma_1\colon [0,1] \to \mathbb{C} - \{0\}$ have the same winding number.

123. Let $n \geq 2$ and let X be the union of two n-dimensional spheres with a point in common. Show that X is simply connected.

124. Find a continuous function $f\colon S^1 \to S^1$ that fails to have a fixed point.

Some Hints

118. Letting $r\colon B^2 \to X$ be the retraction and $i\colon X \to B^2$ be inclusion, you have maps

$$B^2 \xrightarrow{r} X \xrightarrow{f} X \xrightarrow{i} B^2.$$

Apply Brouwer's fixed point theorem to the composition of these maps.

119. You need to find a fixed point for the function defined by

$$f(x,y) = \left(\frac{(x^2+x)\cos t - y^2 \sin t}{5}, \frac{(x^2-1)\sin t + y^3 \cos t}{5} \right).$$

Show that this function maps B^2 to itself.

120. Find the lifting $\widetilde{\gamma}\colon [0,1] \to \mathbb{C}$ by solving the equations $e^{\widetilde{\gamma}(t)} = \gamma(t)$ and $\widetilde{\gamma}(0) = 0$.

121. Show that γ_1 is path homotopic to the loop of the previous problem.

122. Path homotopic loops have liftings that end at the same point.

123. Write X as the union of two open sets and then use Van Kampen's theorem. Neither of the spheres is open in X, by the way.

124. Think of a function that maps each point to the "diametrically opposite" point.