1. Find the eigenvalues and the eigenvectors of the matrix

\[ A = \begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix}. \]

2. Is the following matrix diagonalisable? Why or why not?

\[ A = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}. \]

3. Find a matrix \( A \) that has \( v_1 \) as an eigenvector with eigenvalue \( \lambda_1 = 2 \) and \( v_2 \) as an eigenvector with eigenvalue \( \lambda_2 = 5 \) when

\[ v_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \]

4. Two square matrices \( A, C \) are said to be similar, if \( C = B^{-1}AB \) for some invertible matrix \( B \). Show that similar matrices have the same characteristic polynomial and also the same eigenvalues. Hint: one has \( C - \lambda I = B^{-1}(A - \lambda I)B \).