Linear algebra II
Tutorial problems #1

1. Find the eigenvalues and the eigenvectors of the matrix

$$A = \begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix}.$$

2. Is the following matrix diagonalisable? Why or why not?

$$A = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}.$$

3. Find a matrix A that has v_1 as an eigenvector with eigenvalue $\lambda_1 = 2$ and v_2 as an eigenvector with eigenvalue $\lambda_2 = 5$ when

$$v_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \qquad v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

4. Two square matrices A, C are said to be similar, if $C = B^{-1}AB$ for some invertible matrix B. Show that similar matrices have the same characteristic polynomial and also the same eigenvalues. Hint: one has $C - \lambda I = B^{-1}(A - \lambda I)B$.