MA121 Tutorial Problems #4

1. Suppose f is continuous on $[a,b]$. Show that there exists some $c \in (a,b)$ such that

$$\int_a^b f(t)\,dt = (b-a)\cdot f(c).$$

As a hint, apply the mean value theorem to the function $F(x) = \int_a^x f(t)\,dt$.

2. Suppose that $\sin x$ and $\cos x$ are two functions with the following properties:

$$(\sin x)' = \cos x, \qquad (\cos x)' = -\sin x, \qquad \sin 0 = 0, \qquad \cos 0 = 1.$$

Show that $(\sin x)^2 + (\cos x)^2$ must be constant, namely $(\sin x)^2 + (\cos x)^2 = 1$.

3. Compute each of the following integrals:

$$\int \frac{\sin(1/x)}{x^2}\,dx, \qquad \int (x+1)(x+2)^5\,dx, \qquad \int \frac{x}{\sqrt{x+1}}\,dx, \qquad \int xe^x\,dx.$$

4. Compute each of the following integrals:

$$\int (\sin x)^3\,dx, \qquad \int \frac{x}{e^x}\,dx, \qquad \int e^{\sqrt{x}}\,dx, \qquad \int \frac{\log x}{x^2}\,dx.$$

- These are all practice problems, not a homework assignment.
- However, part of your next homework assignment will be based on these problems.
- In case you get stuck, some hints are provided on the other page of this sheet.

Hints and comments

1. According to the mean value theorem, there exists some $c \in (a, b)$ such that

$$\frac{F(b) - F(a)}{b - a} = F'(c).$$

Simplify this equation by noting that $F(a) = 0$ and that $F'(x) = f(x)$ for all x.

2. Let $f(x) = (\sin x)^2 + (\cos x)^2$. Check that $f'(x) = 0$ and note that $f(0) = 1$.

3a. Use the substitution $u = 1/x = x^{-1}$, which gives $du = -x^{-2}\,dx$.

3b. Use the substitution $u = x + 2$. Split the resulting integral into two parts.

3c. Use the substitution $u = x + 1$. Split the resulting integral into two parts.

3d. Use either integration by parts or tabular integration.

4a. Write $\sin^3 x = (1 - \cos^2 x) \sin x$ and then split the given integral into two parts.

4b. You need to integrate xe^{-x}. Use either integration by parts or tabular integration.

4c. Use the substitution $u = \sqrt{x}$. This gives $x = u^2$ so that $dx = 2u\,du$ and

$$\int e^{\sqrt{x}}\,dx = 2 \int ue^u\,du.$$

Note that you have already computed the rightmost integral; see question 3.

4d. Integrate by parts using $u = \log x$ and $dv = x^{-2}\,dx$.