MA121 Tutorial Problems #3

1. Evaluate each of the following limits:

$$\lim_{x\to+\infty} \frac{6x^2-5}{2-3x^2}, \qquad \lim_{x\to-\infty} \frac{6x^3-5x^2+2}{1-3x+x^4}.$$

2. Find the minimum value of $f(x) = (2x^2-5x+2)^3$ over the closed interval $[0,1]$.

3. Show that $\log x \leq x-1$ for all $x > 0$.

4. Compute each of the following limits:

$$L_1 = \lim_{x\to\infty} \frac{\log(x^2+1)}{x}, \qquad L_2 = \lim_{x\to 1} \frac{x^3+x^2-5x+3}{x^3-x^2-x+1}.$$

5. Suppose that $x > y > 0$. Using the mean value theorem or otherwise, show that

$$1 - \frac{y}{x} < \log x - \log y < \frac{x}{y} - 1.$$

- These are all practice problems, not a homework assignment.
- However, part of your next homework assignment will be based on these problems.
- In case you get stuck, some hints are provided on the other page of this sheet.

Some hints

1. To compute the limit of a rational function as $x \to \pm\infty$, divide both the numerator and the denominator by the highest power of x that appears in the denominator.

2. Since f is continuous on a closed interval, it suffices to check the endpoints, the points at which f' does not exist and the points at which f' is equal to zero. In this case,

$$f'(x) = 3(2x^2 - 5x + 2)^2 \cdot (4x - 5).$$

Make sure that you only consider points which lie in the given closed interval.

3. Check that the maximum value of $f(x) = \log x - x + 1$ is given by $f(1) = 0$.

4. Those are ∞/∞ and $0/0$ limits, so one may use L'Hôpital's rule.

5. First of all, note that the given inequality can also be written in the form

$$\frac{1}{x} < \frac{\log x - \log y}{x - y} < \frac{1}{y}.$$

According to the mean value theorem, one must have

$$f'(c) = \frac{f(x) - f(y)}{x - y}$$

for some $c \in (y, x)$. Use this fact for the case that $f(x) = \log x$.