MA121 Tutorial Problems #1

- 1. Make a table listing the min, inf, max and sup of each of the following sets; write DNE for all quantities which fail to exist. You need not justify any of your answers.
 - (a) $A = \left\{ n \in \mathbb{N} : \frac{1}{n} > \frac{1}{3} \right\}$ (d) $D = \left\{ x \in \mathbb{R} : x < y \text{ for all } y > 0 \right\}$ (b) $B = \left\{ x \in \mathbb{R} : x > 1 \text{ and } 2x \le 5 \right\}$ (e) $E = \left\{ x \in \mathbb{R} : x > y \text{ for all } y > 0 \right\}$
 - (c) $C = \{x \in \mathbb{Z} : x > 1 \text{ and } 2x \le 5\}$
- **2.** Let $x \in \mathbb{R}$ be such that x > -1. Show that $(1 + x)^n \ge 1 + nx$ for all $n \in \mathbb{N}$.
- **3.** Letting $f(x) = x^2 x$ for all $x \in \mathbb{R}$, show that $\inf f(x) = -1/4$.
- **4.** Let A, B be nonempty subsets of \mathbb{R} such that $\sup A < \sup B$. Show that there exists an element $b \in B$ which is an upper bound of A.
- **5.** Given any real number $x \neq 1$, show that

$$1 + x + \ldots + x^n = \frac{1 - x^{n+1}}{1 - x} \quad \text{for all } n \in \mathbb{N}.$$

- These are all practice problems, not a homework assignment.
- However, part of your next homework assignment will be based on these problems.
- In case you get stuck, some hints are provided on the other page of this sheet.

Some hints

- 1a. This set contains only two elements.
- **1b.** This set is actually an interval, namely B = (1, 5/2].
- **1c.** This set contains only one element.
- 1d. The elements of this set must be smaller than all positive reals. Draw the real line and ask yourselves: where are the positive reals? where can the elements of the set be? which are the elements of the set? why am I reading this hint?
- **1e.** The elements of this set must be bigger than all positive reals. Is there any real number with this property?
- **2.** Use induction: prove when n = 1, assume for some $n \in \mathbb{N}$ and then prove for n + 1.
- **3.** Complete the square to show that $f(x) \ge -1/4$ for all x and then argue that -1/4 is the minimum value attained by the function; why is it attained? Once a minimum is known to exist, an infimum also does and the two are equal.
- 4. This problem is not as hard as it may seem. You know that $\sup A < \sup B$ and you want to somehow use this fact to get started. Note that a number "smaller than the least upper bound" cannot be an upper bound. In this case, $\sup A$ is not an upper bound of B, so some element $b \in B$ is such that $b > \sup A$. The rest of the argument should be easy.
- 5. Use induction: prove when n = 1, assume for some $n \in \mathbb{N}$ and then prove for n + 1.