UNIVERSITY OF DUBLIN

XMA1211

Trinity Term 2008

TRINITY COLLEGE

Faculty of Engineering, Mathematics and Science

SCHOOL OF MATHEMATICS

JF Mathematics JF Theoretical Physics JF TSM, SF TSM

Course 121

Tuesday, May 27

Goldsmith Hall

9:30-12:30

Dr. P. Karageorgis

Attempt all questions. All questions are weighted equally. Non-programmable calculators are permitted for this examination. Log tables are available from the invigilators, if required.

- 1. Let A, B be nonempty subsets of \mathbb{R} such that $\inf A < \inf B$. Show that there exists an element $a \in A$ which is a lower bound of B.
- 2. Let f be the function defined by

$$f(x) = \left\{ \begin{array}{ll} 3x+1 & \text{if } x \in \mathbb{Q} \\ 6-2x & \text{if } x \notin \mathbb{Q} \end{array} \right\}.$$

Show that f is continuous at y = 1.

- 3. Show that $2e \cdot x^2 \log x \ge -1$ for all x > 0. Here, e is the usual constant $e \approx 2.718$.
- 4. Compute each of the following integrals:

$$\int \frac{4x^2 - 5x + 2}{x^3 - x^2} \, dx, \qquad \int \sin^3 x \, dx.$$

5. Using the mean value theorem, or otherwise, show that

$$(b-a)e^a < e^b - e^a < (b-a)e^b$$
 whenever $a < b$.

6. Test each of the following series for convergence:

$$\sum_{n=1}^{\infty} \frac{2^n + 4^n}{3^n + 5^n}, \qquad \sum_{n=1}^{\infty} \sin(1/n^2).$$

7. Suppose f, g are integrable on [a, b] with $f(x) \leq g(x)$ for all $x \in [a, b]$. Show that

$$\int_{a}^{b} f(x) \, dx \le \int_{a}^{b} g(x) \, dx.$$

- 8. Letting $f(x, y) = \log(x^2 + y^2)$, find the rate at which f is changing at the point (2, 3)in the direction of the vector $\mathbf{v} = \langle 3, 4 \rangle$.
- 9. Classify the critical points of the function defined by $f(x,y) = x^2 + 2y^2 x^2y$.
- 10. Compute the double integrals (I have included several of those for practice)

$$\int_0^{\pi} \int_x^{\pi} \frac{\sin y}{y} \, dy \, dx, \quad \int_0^1 \int_y^1 x^2 e^{xy} \, dx \, dy, \quad \int_0^2 \int_{x^2}^4 x e^{y^2} \, dy \, dx.$$

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