# UNIVERSITY OF DUBLIN

XMA1213

## TRINITY COLLEGE

Faculty of Engineering, Mathematics
and Science

school of mathematics

**JF Maths, JF TP** **Hilary Term 2009**
**JF TSM**

Course 121

Monday, March 9 Regent House 14:00 – 16:00

Dr. P. Karageorgis

Attempt all questions. All questions are weighted equally.
Log tables are available from the invigilators, if required.

**1.** Compute each of the following integrals:

$$\int \frac{3x+5}{x^3-x}\,dx, \qquad \int x\cos x\,dx.$$

**2.** Let $f$ be defined by $f(x) = \left\{ \begin{array}{ll} 1 & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{array} \right\}$. Show that $f$ is integrable on $[0,1]$.

**3.** Define a sequence $\{a_n\}$ by setting $a_1 = 4$ and

$$a_{n+1} = \frac{1}{5-a_n} \quad \text{for each } n \geq 1.$$

Show that $0 \leq a_{n+1} \leq a_n \leq 4$ for each $n \geq 1$, use this fact to conclude that the sequence converges and then find its limit.

**4.** Compute each of the following limits:

$$\lim_{x\to 0} \frac{e^x - x - 1}{x^2}, \qquad \lim_{x\to\infty} x\sin(1/x).$$

**5.** Test each of the following series for convergence:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\,n^2}{1+n^4}, \qquad \sum_{n=1}^{\infty} \left(\frac{2n}{1+3n}\right)^n.$$

**6.** Find the radius of convergence of the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} \cdot x^n.$$

**7.** Suppose $f$ is a differentiable function such that $f'(x) = 2x \cdot f(x)$ for all $x \in \mathbb{R}$. Show that there exists some constant $C$ such that $f(x) = Ce^{x^2}$ for all $x \in \mathbb{R}$.

**8.** Suppose that $f$ is a function with

$$|f(x) - f(y)| \leq |x-y|^2 \quad \text{for all } x, y \in \mathbb{R}.$$

Using the limit definition of the derivative, show that $f$ is actually constant.