UNIVERSITY OF DUBLIN

XMA1212

TRINITY COLLEGE

Faculty of Engineering, Mathematics and Science

SCHOOL OF MATHEMATICS

JF Maths, JF TP JF TSM Michaelmas Term 2008

Course 121

Monday, December 8

Luce Hall

14:00 - 16:00

Dr. P. Karageorgis

Attempt all questions. All questions are weighted equally. You may use non-programmable calculators, but you may not use log tables.

- 1. Make a table listing the min, inf, max and sup of each of the following sets; write DNE for all quantities which fail to exist. You need not justify any of your answers.
 - (a) $A = \{n \in \mathbb{N} : n 1 \in \mathbb{N}\}$ (c) $C = \{x \in \mathbb{R} : |x| < y \text{ for all } y > 0\}$

(b)
$$B = \{x \in \mathbb{R} : 2x \le 5\}$$
 (d) $D = \{x \in \mathbb{R} : |x+1| < 1\}$

2. Let f be the function defined by

$$f(x) = \left\{ \begin{array}{ll} \frac{4x^3 - 7x - 3}{2x - 3} & \text{if } x \neq 3/2 \\ 10 & \text{if } x = 3/2 \end{array} \right\}.$$

Show that f is continuous at y = 3/2. As a hint, one may avoid the ε - δ definition here.

- 3. Show that there exists some 0 < x < 1 such that $(x^2 2x + 3)^3 = (2x^2 x + 1)^4$.
- 4. Find the maximum value of $f(x) = x(7 x^2)^3$ over the closed interval [-1, 3].
- 5. Suppose that f is a differentiable function such that

$$f'(x) = \frac{1}{1+x^2} \quad \text{for all } x \in \mathbb{R}.$$

Show that f(x) + f(1/x) = 2f(1) for all x > 0.

6. Let f be the function defined by

$$f(x) = \left\{ \begin{array}{ll} 2 - 3x & \text{if } x \le 2\\ 4 - 5x & \text{if } x > 2 \end{array} \right\}.$$

Show that f is discontinuous at y = 2.

- 7. Let A be a nonempty subset of \mathbb{R} that has an upper bound and let $\varepsilon > 0$ be given. Show that there exists some element $a \in A$ such that $\sup A - \varepsilon < a \leq \sup A$.
- 8. Show that the polynomial $f(x) = x^4 2x^3 + x^2 1$ has exactly one root in (1, 2).

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