MA121, Homework #4
due Thursday, Jan. 22 in class

1. Let f be the function defined by

$$f(x) = \left\{ \begin{array}{ll} 0 & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{array} \right\}.$$

Show that f is integrable on $[0, 1]$.

2. Suppose f, g are both integrable on $[a, b]$ and $f(x) \leq g(x)$ for all $x \in [a, b]$. Show that

$$\int_a^b f(x)\, dx \leq \int_a^b g(x)\, dx.$$

3. Show that there exists a unique function f which is defined for all $x \in \mathbb{R}$ and satisfies

$$f'(x) = e^{-x^2}, \qquad f(0) = 0.$$

4. Let f be the function of the previous exercise. Show that f is increasing and that

$$0 \leq f(x) \leq x \quad \text{for all } x \geq 0.$$

Hint: to prove the inequalities, note that $0 \leq e^{-x^2} \leq 1$ and then use exercise 2.

- You are going to work on these problems during your Friday tutorials.
- When writing up solutions, write legibly and coherently. Use words, not just symbols.
- Write both your name and your tutor's name on the first page of your homework.
- Your tutor's name is Thomas, if you are a TP student; otherwise, it is Pete.
- Your solutions may use any of the results stated in class (but nothing else).
- NO LATE HOMEWORK WILL BE ACCEPTED.