## MA121, Homework #3

due Thursday, Nov. 27 in class

- 1. Find the maximum value of  $f(x) = 3x^4 16x^3 + 18x^2$  over the closed interval [0,3].
- 2. Show that the polynomial  $f(x) = x^5 3x + 1$  has three roots in the interval (-2, 2). As a hint, you might wish to compute the values of f at the points  $\pm 2$ ,  $\pm 1$  and 0.
- **3.** Show that the polynomial  $f(x) = x^5 5x + 1$  has exactly one root in (1, 2).
- **4.** Suppose f is a differentiable function with  $|f'(x)| \leq 1$  for all  $x \in \mathbb{R}$ . Show that

$$|f(x) - f(y)| \le |x - y|$$
 for all  $x, y \in \mathbb{R}$ .

*Hint: this is obvious when* x = y*; use the mean value theorem for the case*  $x \neq y$ *.* 

5. Letting  $f(x) = x^5 - 5x + 1$  for all  $x \in \mathbb{R}$ , compute each of the following:

$$\max_{-2 < x < 0} f(x), \qquad \max_{0 < x \le 2} f(x), \qquad \min_{-1 \le x \le 2} f(x).$$

- You are going to work on these problems during your Friday tutorials.
- When writing up solutions, write legibly and coherently. Use words, not just symbols.
- Write both your name and your tutor's name on the first page of your homework.
- Your tutor's name is Stephen, if you are a TP student; otherwise, it is Pete.
- Your solutions may use any of the results stated in class (but nothing else).
- NO LATE HOMEWORK WILL BE ACCEPTED.