

MA121, Homework #2
due Thursday, Nov. 6 in class

1. Make a table listing the min, inf, max and sup of each of the following sets; write DNE for all quantities which fail to exist. You need not justify any of your answers.

(a) $A = \{n \in \mathbb{N} : \frac{n}{n+1} < \frac{1}{2}\}$

(d) $D = \{x \in \mathbb{R} : |x| < y \text{ for all } y > 0\}$

(b) $B = \{x \in \mathbb{Z} : x > 2 \text{ and } 2x \leq 9\}$

(e) $E = \{x \in \mathbb{R} : |x - 2| < 3\}$

(c) $C = \{x \in \mathbb{R} : x < y \text{ for all } y \in \mathbb{N}\}$

2. Let f be a function such that $f(1) = 5$ and $f(n+1) = 2f(n) + 1$ for all $n \in \mathbb{N}$. Use induction to show that we actually have $f(n) = 3 \cdot 2^n - 1$ for all $n \in \mathbb{N}$.
3. Suppose f, g are functions with $f(x) \leq g(x)$ for all x . Show that $\sup f(x) \leq \sup g(x)$.
4. Evaluate the limit

$$L = \lim_{x \rightarrow 1} \frac{x^3 + 3x^2 - 9x + 5}{(x-1)^2}.$$

5. Show that the function f defined by

$$f(x) = \begin{cases} 2x - 7 & \text{if } x \leq 3 \\ 8 - 3x & \text{if } x > 3 \end{cases}$$

is continuous at $y = 3$.

- You are going to work on these problems during your Friday tutorials.
- When writing up solutions, write legibly and coherently. Use words, not just symbols.
- Write both your name and your tutor's name on the first page of your homework.
- Your tutor's name is Stephen, if you are a TP student; otherwise, it is Pete.
- Your solutions may use any of the results stated in class (but nothing else).
- NO LATE HOMEWORK WILL BE ACCEPTED.