UNIVERSITY OF DUBLIN

XMA1211

TRINITY COLLEGE

Faculty of Engineering, Mathematics and Science

SCHOOL OF MATHEMATICS

JF Maths, JF TP JF TSM Trinity Term 2009

Course 121

Tuesday, May 26

Upper Luce Hall 9:30 – 12:30

Dr. P. Karageorgis & Dr. J. Stalker

Attempt all questions. All questions are weighted equally. Log tables are available from the invigilators, if required.

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- 1. Show that there exists some 0 < x < 1 such that $4x^3 + 3x = 2x^2 + 2$.
- 2. Suppose f is a differentiable function with $|f'(x)| \leq 1$ for all $x \in \mathbb{R}$. Show that

$$|f(x) - f(y)| \le |x - y|$$
 for all $x, y \in \mathbb{R}$.

- 3. Suppose f is a differentiable function with $f'(x) = f(x) + e^x$ for all $x \in \mathbb{R}$. Show that there exists some constant C such that $f(x) = xe^x + Ce^x$ for all $x \in \mathbb{R}$.
- 4. Compute each of the following integrals:

$$\int \frac{3x+2}{(x+1)^2} \, dx, \qquad \int \log(x^2 - 1) \, dx$$

- 5. Show that $\log x \le x 1$ for all x > 0.
- 6. Test each of the following series for convergence:

$$\sum_{n=1}^{\infty} \frac{e^{1/n}}{n}, \qquad \sum_{n=1}^{\infty} \frac{1}{n^n}, \qquad \sum_{n=1}^{\infty} \frac{(-1)^n \sin n}{n^2}.$$

7. Find the maximum value of $f(\boldsymbol{x},\boldsymbol{y})=\boldsymbol{x}\boldsymbol{y}^2$ over the closed unit disk

$$D = \{ (x, y) \in \mathbb{R}^2 : x^2 + y^2 \le 1 \}.$$

8. Compute the double integral

$$\int_0^4 \int_{\sqrt{y}}^2 \cos(x^3) \, dx \, dy$$

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