UNIVERSITY OF DUBLIN

XMA1213

TRINITY COLLEGE

Faculty of Engineering, Mathematics and Science

SCHOOL OF MATHEMATICS

JF Maths, JF TP JF TSM Hilary Term 2009

Course 121

Monday, March 9

Regent House

14:00 - 16:00

Dr. P. Karageorgis

Attempt all questions. All questions are weighted equally. Log tables are available from the invigilators, if required.

1. Compute each of the following integrals:

$$\int \frac{5x-1}{x^3-x} \, dx, \qquad \int (\log x)^2 \, dx.$$

2. Let $a, b \in \mathbb{R}$ be some fixed numbers and consider the function f defined by

$$f(x) = \left\{ \begin{array}{ll} a & \text{ if } x \in \mathbb{Q} \\ b & \text{ if } x \notin \mathbb{Q} \end{array} \right\}.$$

Show that f is integrable on [0,1] if and only if a = b.

- Define a sequence {a_n} by setting a₁ = 2 and a_{n+1} = √4a_n + 3 for each n ≥ 1. Show that 2 ≤ a_n ≤ a_{n+1} ≤ 5 for each n ≥ 1, use this fact to conclude that the sequence converges and then find its limit.
- 4. Test each of the following series for convergence:

$$\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}, \qquad \sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}, \qquad \sum_{n=1}^{\infty} \frac{n^e}{e^n}.$$

5. Find the radius of convergence of the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} \cdot x^n.$$

6. Let f be the function defined by the formula

$$f(x) = \int_0^x e^{t^2} dt$$

Find the Taylor series of f (around the point x = 0). For which values of x does this series converge? As a hint, you need only integrate the Taylor series for e^{t^2} .

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