UNIVERSITY OF DUBLIN

TRINITY COLLEGE

FACULTY OF ENGINEERING, MATHEMATICS
AND SCIENCE

SCHOOL OF MATHEMATICS

JF Maths, JF TP   **Michaelmas Term 2008**
JF TSM

COURSE 121

Monday, December 8   Luce Hall   14:00 − 16:00

Dr. P. Karageorgis

Attempt all questions. All questions are weighted equally.
You may use non-programmable calculators, but you may not use log tables.

1. Show that the set $A = \{x \in \mathbb{R} : |x - 2| < 1\}$ is such that $\sup A = 3$.

2. Let f be the function defined by

$$f(x) = \left\{ \begin{array}{ll} 2x - 1 & \text{if } x \leq 2 \\ 3x - 2 & \text{if } x > 2 \end{array} \right\}.$$

   Show that f is not continuous at $y = 2$.

3. Show that the polynomial $f(x) = x^3 - 4x^2 - 3x + 1$ has <u>exactly</u> one root in $(0, 2)$.

4. Find the maximum value of $f(x) = x(7 - x^2)^3$ over the closed interval $[0, 3]$.

5. Show that $xe^x \geq e^x - 1$ for all $x \in \mathbb{R}$.

6. Suppose that $x > y > 0$. Using the mean value theorem or otherwise, show that

$$1 - \frac{y}{x} < \log \frac{x}{y} < \frac{x}{y} - 1.$$