UNIVERSITY OF DUBLIN

XMA1213

TRINITY COLLEGE

FACULTY OF SCIENCE

SCHOOL OF MATHEMATICS

JF Maths, JF TP **Hilary Term 2008**
JF TSM, SF TSM

COURSE 121

Monday, March 10 Regent House 14:00 – 17:00

Dr. P. Karageorgis

Attempt all questions. All questions are weighted equally.
You may use non-programmable calculators, but you may not use log tables.

1. Compute each of the following integrals:

$$\int \frac{3x-1}{x^3-x}\,dx, \qquad \int x\log x\,dx.$$

2. Suppose f, g are integrable on $[a,b]$ with $f(x) \leq g(x)$ for all $x \in [a,b]$. Show that

$$\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx.$$

3. Define a sequence $\{a_n\}$ by setting $a_1 = 1$ and

$$a_{n+1} = \sqrt{3a_n - 1} \quad \text{for each } n \geq 1.$$

Show that $1 \leq a_n \leq a_{n+1} \leq 3$ for each $n \geq 1$, use this fact to conclude that the sequence converges and then find its limit.

4. Compute each of the following limits:

$$\lim_{x\to 1} \frac{x^3-5x^2+7x-3}{x^3-4x^2+5x-2}, \qquad \lim_{x\to\infty} x\sin(1/x).$$

5. Test each of the following series for convergence:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\,e^{1/n}}{n}, \qquad \sum_{n=1}^{\infty} \log\left(1+\frac{1}{n}\right).$$

6. Find the radius of convergence of the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}\cdot x^n.$$

7. Suppose f is a differentiable function such that $f'(x) = f(x) + e^x$ for all $x \in \mathbb{R}$. Show that there exists some constant C such that $f(x) = xe^x + Ce^x$ for all $x \in \mathbb{R}$.

8. Use the formula for a geometric series to show that

$$\sum_{n=0}^{\infty} n^2x^n = \frac{x(1+x)}{(1-x)^3} \quad \text{whenever } |x| < 1.$$