UNIVERSITY OF DUBLIN

TRINITY COLLEGE

FACULTY OF SCIENCE

SCHOOL OF MATHEMATICS

JF Mathematics JF Theoretical Physics JF TSM, SF TSM

Hilary Term 2007

Course 121

Monday, March 12

Regent House

14:00 - 17:00

Dr. P. Karageorgis

Attempt all questions. All questions are weighted equally.

XMA1213

- 1. Find the minimum value of $f(x) = x^4 + 4x^3 8x^2 + 2$ over the closed interval [0, 2].
- 2. Determine the minimum and maximum values attained by $f(x) = \frac{2x+1}{x^2+2}$.
- 3. Compute the following limit:

$$\lim_{x \to 1} \frac{x^3 + 4x^2 + x - 6}{x^3 - x^2 - 4x + 4}$$

4. Evaluate each of the following integrals:

$$\int \frac{4x^2 + x + 2}{x^3 + x} \, dx, \qquad \int x \cdot e^x \, dx.$$

5. Suppose that f is a differentiable function such that

$$f'(x) = \cos x \cdot f(x)$$
 for all $x \in \mathbb{R}$.

Show that there exists some constant C such that $f(x) = Ce^{\sin x}$ for all $x \in \mathbb{R}$.

6. Assuming that f is continuous on [a,b] with $\int_a^b f(t) dt = 0$, show that

$$f(c) = 0$$
 for some $c \in (a, b)$.

As a hint, apply the mean value theorem to the function $F(x) = \int_a^x f(t) dt$.

- 7. Let f be a non-negative function which is integrable on [0,1] with f(x) = 0 for all $x \in \mathbb{Q}$. Show that $\int_0^1 f(x) dx = 0$.
- 8. Let f be the function defined by

$$f(x) = \left\{ \begin{array}{rr} x^3 \cdot \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{array} \right\}.$$

Using the limit definition of a derivative, show that f'(0) = 0. You may use the fact that

$$-1 \le \sin(1/x) \le 1$$
 for all $x \ne 0$.

© UNIVERSITY OF DUBLIN 2007