MAU11201 – Calculus
Homework #1
due Thursday, Sep. 19

1. Find the domain and the range of the function f which is defined by

$$f(x) = \frac{2-3x}{7-2x}.$$

2. Show that the function $f\colon (0,1) \to (0,2)$ is bijective in the case that

$$f(x) = \frac{4x}{3-x}.$$

3. Find the domain and the range of the function f which is defined by

$$f(x) = \sqrt{4-\sqrt{x}}.$$

4. Express the following polynomials as the product of linear factors.

$$f(x) = 3x^3 + 4x^2 - 5x - 2, \qquad g(x) = x^3 - \frac{7x^2}{6} + \frac{1}{6}.$$

5. Determine all angles $0 \leq \theta \leq 2\pi$ such that $4\sin^2\theta + 4\sin\theta = 3$.

- This assignment is due by Thursday noon, either in class or else in my office.
- Write your name and also your course on the first page of your homework.
- NO LATE HOMEWORK WILL BE ACCEPTED.

MAU11201 – Calculus
Homework #2
due Thursday, Sep. 26

1. Determine the inverse function f^{-1} in each of the following cases.

$$f(x) = \frac{1}{3}\log_2(2x-6) - 1, \qquad f(x) = \frac{7\cdot 5^x - 3}{4\cdot 5^x + 2}.$$

2. Simplify each of the following expressions.

$$\cos\left(\sin^{-1} x\right), \qquad \cos\left(\tan^{-1} x\right), \qquad \log_3(54) - 3\log_3(18) + \log_3(36).$$

3. Use the ε-δ definition of limits to compute $\lim_{x\to 3} f(x)$ in the case that

$$f(x) = \left\{ \begin{array}{ll} 3x-4 & \text{if } x \leq 3 \\ 4x-7 & \text{if } x > 3 \end{array} \right\}.$$

4. Compute each of the following limits.

$$L = \lim_{x\to 1} \frac{3x^3 - 7x^2 + 6x - 2}{x-1}, \qquad M = \lim_{x\to 2} \frac{2x^3 - 7x^2 + 4x + 4}{(x-2)^2}.$$

5. Use the ε-δ definition of limits to compute $\lim_{x\to 2}(3x^2 - 4x + 7)$.

- This assignment is due by Thursday noon, either in class or else in my office.
- Write your name and also your course on the first page of your homework.
- NO LATE HOMEWORK WILL BE ACCEPTED.

MAU11201 – Calculus
Homework #3
due Thursday, Oct. 3

1. Show that there exists a real number $0 < x < \pi/2$ that satisfies the equation

$$x \sin x + x \cos x = 1.$$

2. For which values of a, b is the function f continuous at the point $x = 3$? Explain.

$$f(x) = \left\{ \begin{array}{cl} 4x^2 + ax + b & \text{if } x < 3 \\ a + b - 2 & \text{if } x = 3 \\ 2x^3 - bx + a & \text{if } x > 3 \end{array} \right\}.$$

3. Show that $f(x) = 2x^5 - 3x^3 - 5x + 1$ has three roots in the interval $(-2, 2)$. Hint: you need only consider the values that are attained by f at the points ± 2, ± 1 and 0.

4. Compute each of the following limits.

$$L = \lim_{x \to +\infty} \frac{2x^4 - 4x^2 + 5}{3x^4 - 7x + 2}, \qquad M = \lim_{x \to 3^-} \frac{x^3 - 5x + 4}{x^3 - 8x - 3}.$$

5. Use the definition of the derivative to compute $f'(x_0)$ in each of the following cases.

$$f(x) = (3x + 1)^2, \qquad f(x) = (x^2 - 1)^2.$$

- This assignment is due by Thursday noon, either in class or else in my office.
- Write your name and also your course on the first page of your homework.
- NO LATE HOMEWORK WILL BE ACCEPTED.

MAU11201 – Calculus
Homework #4
due Thursday, Oct. 10

1. Compute the derivative $y' = \frac{dy}{dx}$ in each of the following cases.

$$y = \ln(\tan x) + 2(\sec x)^5, \qquad y = \tan^{-1}(\sin(2x)).$$

2. Compute the derivative $y' = \frac{dy}{dx}$ in the case that $y^2 \cos x + x^3 e^y = x^2 y^3$.

3. Compute the derivative $f'(x_0)$ in the case that

$$f(x) = \frac{(x^3+2)^3 \cdot e^{4x} \cdot \cos(5\tan x)}{\sqrt{x^3+1}}, \qquad x_0 = 0.$$

4. Show that the derivative of the inverse tangent function is given by

$$\left(\tan^{-1} x\right)' = \frac{1}{1+x^2}.$$

5. Compute the derivative $f'(2)$ in the case that $x^2 e^{f(x)} + 3xe^{2f(x)} = 2$ for all x.

Hint: write $x^2 e^y + 3xe^{2y} = 2$ for simplicity. You will need to determine the values of y and y' when $x = 2$.

- This assignment is due by Thursday noon, either in class or else in my office.
- Write your name and also your course on the first page of your homework.
- NO LATE HOMEWORK WILL BE ACCEPTED.

MAU11201 – Calculus
Homework #5
due Thursday, Oct. 17

1. Show that $f(x) = 2x^3 - 3x^2 - 4x + 1$ has exactly one root in $(0, 1)$.

2. Compute each of the following limits.

$$L_1 = \lim_{x \to 2} \frac{3x^2 - 5x - 2}{2x^2 - 7x + 6}, \qquad L_2 = \lim_{x \to \infty} \frac{(\ln x)^2}{x}, \qquad L_3 = \lim_{x \to 0^+} (e^{3x} + \sin x)^{2/x}.$$

3. On which intervals is f increasing? On which intervals is it concave up?

$$f(x) = \ln(4x^2 + 1).$$

4. Find the intervals on which f is increasing/decreasing and the intervals on which f is concave up/down. Use this information to sketch the graph of f.

$$f(x) = \frac{x}{x^2 + 1}.$$

5. Show that the cubic polynomial $f(x) = x^3 + ax^2 + bx + c$ has a unique real root for any given constants a, b, c such that $a^2 < 3b$.

- This assignment is due by Thursday noon, either in class or else in my office.
- Write your name and also your course on the first page of your homework.
- NO LATE HOMEWORK WILL BE ACCEPTED.

MAU11201 – Calculus
Homework #6
due Thursday, Oct. 31

1. Find the global minimum and the global maximum values that are attained by

$$f(x) = 4x^3 + x^2 - 2x - 1, \qquad 0 \leq x \leq 1.$$

2. Find the linear approximation to the function f at the point x_0 in the case that

$$f(x) = \frac{3x^4 - 4x + 2}{x^2 + 3x + 1}, \qquad x_0 = 0.$$

3. Show that $f(x) = x^3 - 4x^2 + 1$ has exactly two roots in $(-1, 1)$ and use Newton's method with $x_1 = \pm 1$ to approximate these roots within two decimal places.

4. A rectangle is inscribed in an equilateral triangle of side length $a > 0$ with one of its sides along the base of the triangle. How large can the area of the rectangle be?

5. A ladder 5m long is resting against a vertical wall. The bottom of the ladder slides away from the wall at the rate of 0.2m/s. How fast is the angle θ between the ladder and the wall changing when the bottom of the ladder lies 3m away from the wall?

Hint: There is a right triangle with sides $x, y, 5$ such that $x^2 + y^2 = 5^2$ and $\tan\theta = x/y$. Determine y' and then θ'.

- This assignment is due by Thursday noon, either in class or else in my office.
- Write your name and also your course on the first page of your homework.
- NO LATE HOMEWORK WILL BE ACCEPTED.

MAU11201 – Calculus
Homework #7
due Thursday, Nov. 7

1. Find the area of the region enclosed by the graphs of $f(x) = 3x^2$ and $g(x) = x + 2$.

2. Compute the volume of a sphere of radius $r > 0$. Hint: one may obtain such a sphere by rotating the upper semicircle $f(x) = \sqrt{r^2 - x^2}$ around the x-axis.

3. Compute the length of the graph of $f(x) = \frac{x^4}{16} + \frac{1}{2x^2}$ over the interval $[1, 3]$.

4. Find both the mass and the centre of mass for a thin rod whose density is given by

$$\delta(x) = x^2 + 2x + 3, \qquad 1 \leq x \leq 2.$$

5. Use the definition of integrals and Riemann sums to compute the value of the limit

$$\lim_{n\to\infty} \left(\frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \ldots + \frac{n}{n^2 + n^2} \right).$$

Hint: express the given sum in the form $\sum_{k=1}^{n} \frac{1}{n} f(\frac{k}{n})$, where $f(x) = \frac{1}{1+x^2}$. The limit of this Riemann sum is an integral.

- This assignment is due by Thursday noon, either in class or else in my office.
- Write your name and also your course on the first page of your homework.
- NO LATE HOMEWORK WILL BE ACCEPTED.

MAU11201 – Calculus
Homework #8
due Thursday, Nov. 14

1. Compute each of the following indefinite integrals.

$$\int \cos\sqrt{x}\,dx, \qquad \int x^2\cdot\sqrt{x+1}\,dx.$$

2. Compute each of the following indefinite integrals.

$$\int \sin^3 x\cdot\cos^2 x\,dx, \qquad \int \tan^4 x\cdot\sec^6 x\,dx.$$

3. Compute each of the following indefinite integrals.

$$\int \frac{x^2}{\sqrt{9-x}}\,dx, \qquad \int \frac{x^2}{\sqrt{9-x^2}}\,dx.$$

4. Compute each of the following indefinite integrals.

$$\int \frac{2x+1}{x^2-3x+2}\,dx, \qquad \int \frac{2+e^x}{3-e^x}\,dx.$$

5. Find the volume of the solid that is obtained by rotating the graph of $f(x) = \sin x$ around the x-axis over the interval $[0,\pi]$.

- This assignment is due by Thursday noon, either in class or else in my office.
- Write your name and also your course on the first page of your homework.
- NO LATE HOMEWORK WILL BE ACCEPTED.

MAU11201 – Calculus
Homework #9
due Thursday, Nov. 21

1. Compute each of the following indefinite integrals.

$$\int \frac{x^2-2x-3}{x^3-x^2}\,dx, \qquad \int \frac{x^3-x^2}{x^2-2x-3}\,dx.$$

2. Compute each of the following indefinite integrals.

$$\int \frac{2+\sqrt{x}}{x+\sqrt{x}}\,dx, \qquad \int \ln(x^2+x)\,dx.$$

3. Use integration by parts and induction to show that

$$\int_0^{\pi/2} \sin^{2n+1} x\,dx = \frac{(2^n\cdot n!)^2}{(2n+1)!} \quad \text{for each integer } n\geq 0.$$

4. Show that each of the following sequences converges.

$$a_n = \sqrt{\frac{4n^2+5}{9n^2+7}}, \qquad b_n = \frac{(-1)^n}{n^2+1}, \qquad c_n = n\tan\frac{2}{n}.$$

5. Define a sequence $\{a_n\}$ by setting $a_1=1$ and $a_{n+1}=2\sqrt{4+a_n}$ for each $n\geq 1$. Show that $1\leq a_n\leq a_{n+1}\leq 8$ for each $n\geq 1$, use this fact to conclude that the sequence converges and then find its limit.

- This assignment is due by Thursday noon, either in class or else in my office.
- Write your name and also your course on the first page of your homework.
- NO LATE HOMEWORK WILL BE ACCEPTED.