MA1125 – Calculus
Homework #1
due Thursday, Sep. 20

1. Find the domain and the range of the function f which is defined by

$$f(x) = \frac{3-2x}{5-3x}.$$

2. Find the domain and the range of the function f which is defined by

$$f(x) = \frac{\sqrt{2x-1}}{x}.$$

3. Show that the function $f\colon (0,1) \to (1,\infty)$ is bijective in the case that

$$f(x) = \frac{1+x}{1-x}.$$

4. Express the following polynomials as the product of linear factors.

$$f(x) = 3x^3 - 2x^2 - 7x - 2, \qquad g(x) = x^3 + x^2 - \frac{7x}{4} + \frac{1}{2}.$$

5. Determine all angles $0 \leq \theta \leq 2\pi$ such that $2\sin^2\theta + 9\sin\theta = 5$.

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- Write your name and course (Maths, TP, TSM) on the first page of your homework.
- NO LATE HOMEWORK WILL BE ACCEPTED.

MA1125 – Calculus
Homework #2
due Thursday, Sep. 27

1. Determine the inverse function f^{-1} in each of the following cases.

$$f(x) = 3 - \log_2(2x - 4), \qquad f(x) = \frac{2 \cdot 7^x + 3}{5 \cdot 7^x + 4}.$$

2. Simplify each of the following expressions.

$$\cos\left(\tan^{-1} x\right), \qquad \sin\left(\cos^{-1} x\right), \qquad \log_2 \frac{4^x + 8^x}{2^x + 4^x}.$$

3. Use the ε-δ definition of limits to compute $\lim_{x\to 2} f(x)$ in the case that

$$f(x) = \left\{ \begin{array}{ll} 2x - 5 & \text{if } x \leq 2 \\ 5 - 3x & \text{if } x > 2 \end{array} \right\}.$$

4. Compute each of the following limits.

$$L = \lim_{x\to 1} \frac{x^3 - 4x^2 + 4x - 1}{x - 1}, \qquad M = \lim_{x\to 1} \frac{3x^3 - 7x^2 + 5x - 1}{(x - 1)^2}.$$

5. Use the ε-δ definition of limits to compute $\lim_{x\to 3} (5x^2 - 6x + 3)$.

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MA1125 – Calculus
Homework #3
due Thursday, Oct. 4

1. Show that there exists a real number $0 < x < \pi$ that satisfies the equation

$$x^2 = \frac{x^2+1}{2+\sin x} + 4.$$

2. For which values of a, b is the function f continuous at the point $x = 2$? Explain.

$$f(x) = \left\{ \begin{array}{cc} 2x^3 - ax^2 + bx & \text{if } x < 2 \\ a^2 + b & \text{if } x = 2 \\ 2x^2 + bx - a & \text{if } x > 2 \end{array} \right\}.$$

3. Show that $f(x) = x^5 - x^2 - 3x + 1$ has three roots in the interval $(-2, 2)$. Hint: you need only consider the values that are attained by f at the points ± 2, ± 1 and 0.

4. Compute each of the following limits.

$$L = \lim_{x \to +\infty} \frac{3x^3 - 2x + 4}{5x^3 - x^2 + 7}, \qquad M = \lim_{x \to 2^-} \frac{x^3 + 5x^2 - 4}{3x^3 - 16x + 8}.$$

5. Use the definition of the derivative to compute $f'(x_0)$ in each of the following cases.

$$f(x) = x^3, \qquad f(x) = 1/x^2, \qquad f(x) = (3x+4)^2.$$

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MA1125 – Calculus
Homework #4
due Thursday, Oct. 11

1. Compute the derivative $y' = \frac{dy}{dx}$ in each of the following cases.

$$y = \tan(e^x) + e^{\sec x}, \qquad y = \cos(\sin^2(\ln x)).$$

2. Compute the derivative $y' = \frac{dy}{dx}$ in the case that $y^2 \sin x + x^2 \sin y = x^2 y$.

3. Compute the derivative $y' = \frac{dy}{dx}$ in each of the following cases.

$$y = e^{\sin x} \cdot \cos(e^x), \qquad y = (x \cdot \tan x)^{\ln x}.$$

4. Compute the derivative $f'(x_0)$ in the case that

$$f(x) = \frac{(x^2+3)^2 \cdot x^{\ln x} \cdot e^{4-4x}}{\sqrt{e^{2x-2}+3}}, \qquad x_0 = 1.$$

5. Compute the derivative $y' = \frac{dy}{dx}$ in the case that

$$y = \tan^{-1} u, \qquad u = \sqrt{2z^3+1}, \qquad z = \frac{x^2-3}{x^2+1}.$$

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MA1125 – Calculus
Homework #5
due Thursday, Oct. 18

1. Show that the polynomial $f(x) = x^3 - 4x^2 - 3x + 1$ has exactly one root in $(0, 2)$.

2. Suppose that $0 < a < b$. Use the mean value theorem to show that

$$1 - \frac{a}{b} < \ln b - \ln a < \frac{b}{a} - 1.$$

3. Compute each of the following limits.

$$L_1 = \lim_{x \to 3} \frac{2x^3 - 8x^2 + 7x - 3}{3x^3 - 8x^2 - x - 6}, \qquad L_2 = \lim_{x \to \infty} \frac{x^2}{e^x}, \qquad L_3 = \lim_{x \to 0} (e^x + x)^{1/x}.$$

4. On which intervals is f increasing? On which intervals is it concave up?

$$f(x) = \frac{x}{x^2 + 3}.$$

5. Find the intervals on which f is increasing/decreasing and the intervals on which f is concave up/down. Use this information to sketch the graph of f.

$$f(x) = \frac{(x-1)^2}{x^2 + 1}.$$

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MA1125 – Calculus
Homework #6
due Thursday, Nov. 1

1. Find the global minimum and the global maximum values that are attained by

$$f(x) = x^3 - 6x^2 + 9x - 5, \qquad 0 \leq x \leq 2.$$

2. If a right triangle has a hypotenuse of length $a > 0$, how large can its area be?

3. A balloon is rising vertically at the rate of 1 m/sec. When it reaches 48m above the ground, a bicycle passes under it moving at 3 m/sec along a flat, straight road. How fast is the distance between the bicycle and the balloon increasing 16 seconds later?

4. Find the linear approximation to the function f at the point x_0 in the case that

$$f(x) = \frac{x^3 - 2x + 4}{x^2 + 2}, \qquad x_0 = 0.$$

5. Show that $f(x) = x^3 - 4x + 1$ has two roots in $(0, 2)$ and use Newton's method with initial guesses $x_1 = 0, 2$ to approximate these roots within two decimal places.

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MA1125 – Calculus
Homework #7
due Thursday, Nov. 8

1. Find the area of the region enclosed by the graphs of $f(x) = 2x^2$ and $g(x) = x + 6$.

2. Compute the volume of a sphere of radius $r > 0$. Hint: one may obtain such a sphere by rotating the upper semicircle $f(x) = \sqrt{r^2 - x^2}$ around the x-axis.

3. Compute the length of the graph of $f(x) = \frac{1}{3}x^{3/2}$ over the interval $[0, 5]$.

4. Find both the mass and the centre of mass for a thin rod whose density is given by

$$\delta(x) = x^3 + 2x^2 + 5x, \qquad 0 \leq x \leq 2.$$

5. A cylindrical tank of radius 2m and height 3m is full with water of density 1000kg/m^3. How much work is needed to pump out the water through a hole in the top?

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MA1125 – Calculus
Homework #8
due Thursday, Nov. 15

1. Compute each of the following indefinite integrals.

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx, \qquad \int x\sqrt{1-x}\,dx.$$

2. Compute each of the following indefinite integrals.

$$\int \sin^3 x \cdot \cos^4 x\,dx, \qquad \int \tan^2 x \cdot \sec^6 x\,dx.$$

3. Compute each of the following indefinite integrals.

$$\int x^3(\ln x)^2\,dx, \qquad \int x^3\sqrt{4-x^2}\,dx.$$

4. Find the area of the region enclosed by the graphs of $f(x) = e^{2x}$ and $g(x) = 3e^x - 2$.

5. Find the volume of the solid that is obtained by rotating the graph of $f(x) = xe^x$ around the x-axis over the interval $[0, 1]$.

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MA1125 – Calculus
Homework #9
due Thursday, Nov. 22

1. Compute each of the following indefinite integrals.

$$\int \frac{x^3 - 3x + 2}{x + 3}\, dx, \qquad \int \frac{x + 3}{x^3 - 3x + 2}\, dx.$$

2. Compute each of the following indefinite integrals.

$$\int \frac{x + 3}{x + \sqrt{x}}\, dx, \qquad \int \frac{e^x + 3}{e^x + 1}\, dx.$$

3. Compute each of the following indefinite integrals.

$$\int \frac{\sin^3 x}{\cos^8 x}\, dx, \qquad \int \frac{3x + 1}{x^2 + 2x + 5}\, dx.$$

4. Show that each of the following sequences converges.

$$a_n = \cos \frac{n^2 + 2}{n^3 + 1}, \qquad b_n = \frac{(-1)^n}{n^2}, \qquad c_n = n \sin \frac{1}{n}.$$

5. Define a sequence $\{a_n\}$ by setting $a_1 = 1$ and $a_{n+1} = \sqrt{2a_n + 1}$ for each $n \geq 1$. Show that $1 \leq a_n \leq a_{n+1} \leq 3$ for each $n \geq 1$, use this fact to conclude that the sequence converges and then find its limit.

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