Fuzzy Logic Notes

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April 25, 2011

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Chapter 1

Introduction

1.1 Computers

Computers aid us with many areas of life from astronomy to banking to rocket science. However, they have their drawbacks. Computer systems cannot satisfactorily manage information flowing across a hospital, for example. The introduction of computer systems for public administration has invariably generated chaos. Computer systems have been found responsible for disasters like flood damage, fire control and so on.

1.2 Problems

So why can't the computers do what we want them to?

- Problems in engineering software: specification, design, and testing.
- Algorithms (the basis of computer programs) cannot deal with partial information, with uncertainty.
- Much of human information processing relies significantly on approximate reasoning.

1.3 Example: Motion Detection

Suppose we create a robot that will react to certain situations, displaying emotions. We will deal with the motion of "intruders" relative to the robot. We wish it to have the following basic emotions:

Perception	Emotion
If the intruder is far away	there is no fear
If the intruder is near	there is no surprise
If the intruder is stationary	there is no fear
If the intruder is moving fast	there is no anger

This is perhaps better expressed as follows:

	Stationary	Fast
Near	Very angry, not surprised, no fear	Not angry, not surprised, very fearful
Far	Very angry, not surprised, no fear	Not angry, very surprised, no fear

However, near and far are quite relative terms, as is fast. We'll consider a more accurate set of emotions:

	Stationary	Slow	Fast
Very Near	VA, NS, NF	A, NS, F	NA, NS, VF
Near	A, NS, NF	NA, NS, F	NA, S, F
Far	VA, NS, NF	A, S, NF	NA, VS, NF

where V means very, N means not, A means angry, S means surprised and F means fearful. We can make the terms of speed and distance more specific still, and this will give us different combinations of emotions.

1.4 Solution

The solution to the above problem of computers is *soft computing*:

> Soft computing differs from conventional (hard) computing in that, unlike hard computing, it is tolerant of imprecision, uncertainty, partial truth, and approximation. In effect, the role model for soft computing is the human mind. The guiding principle of soft computing is: Exploit the tolerance for imprecision, uncertainty, partial truth, and approximation to achieve tractability, robustness and low solution cost.

Soft computing is used as an umbrella term for sub-disciplines of computing, including fuzzy logic and fuzzy control, neural networks based computing and machine learning, and genetic algorithms, together with chaos theory in mathematics.

Chapter 2

Introduction to Fuzzy Logic

Fuzzy logic is being developed as a discipline to meet two objectives:

- As a professional subject dedicated to the building of systems of high utility - for example fuzzy control.
- As a theoretical subject - fuzzy logic is "symbolic logic with a comparative notion of truth developed fully in the spirit of classical logic. It is a branch of many-valued logic based on the paradigm of inference under vagueness."

2.1 Fuzzy Sets

A FUZZY SET is a set whose elements have degrees of membership. Fuzzy sets are an extension of the classical notion of set (known as a CRISP SET). More mathematically, a fuzzy set is a pair (A, μ_A) where A is a set and $\mu_A : A \to [0, 1]$. For all $x \in A$, $\mu_A(x)$ is called the grade of membership of x.

If $\mu_A(x) = 1$, we say that x is FULLY INCLUDED in (A, μ_A), and if $\mu_A(x) = 0$, we say that x is NOT INCLUDED in (A, μ_A). If there exists some $x \in A$ such that $\mu_A(x) = 1$, we say that (A, μ_A) is NORMAL. Otherwise, we say that (A, μ_A) is SUBNORMAL.

2.2 Fuzzy Logic

FUZZY LOGIC is a form of multi-valued logic derived from fuzzy set theory to deal with reasoning that is approximate rather than precise. Fuzzy logic is not a vague logic system, but a system of logic for dealing with vague concepts.

As in fuzzy set theory the set membership values can range (inclusively) between 0 and 1, in fuzzy logic the degree of truth of a statement can range between 0 and 1 and is not constrained to the two truth values true/false as in classic predicate logic.

2.2.1 Examples of Fuzzy Logic

In a Fuzzy Logic washing machine, Fuzzy Logic detects the type and amount of laundry in the drum and allows only as much water to enter the machine as is really needed for the loaded amount. So, less water will heat up quicker - which means less energy consumption. Additional properties:

- **Foam detection**: Too much foam is compensated by an additional rinse cycle.
- **Imbalance compensation**: In the event of imbalance calculate the maximum possible speed, sets this speed and starts spinning.
- **Water level adjustment**: Fuzzy automatic water level adjustment adapts water and energy consumption to the individual requirements of each wash programme, depending on the amount of laundry and type of fabric.

2.2.2 Other Examples

Other examples of uses of Fuzzy Logic are:

- Food cookers
- Taking blood pressure
- Determination of "socio-economic class"
- Cars

Chapter 3

Fuzzy Systems

3.1 Introduction

A <u>Fuzzy System</u> can be contrasted with a conventional (crisp) system in three main ways:

- A <u>Linguistic Variable</u> is defined as a variable whose values are sentences in a natural or artificial language. Thus, "if tall", "not tall", "very tall", "very very tall", etc. are values of *height*, then *height* is a linguistic variable.
- <u>Fuzzy Conditional Statements</u> are expressions of the form "If A THEN B", where A and B have fuzzy meaning, e.g. "If x is small THEN y is large", where small and large are viewed as labels of fuzzy sets.
- A <u>Fuzzy Algorithm</u> is an ordered sequence of instructions which may contain fuzzy assignment and conditional statements, e.g., "x = very small, IF x is small THEN y is large". The execution of such instructions is governed by the compositional rule of inference and the rule of the preponderant alternative.

3.1.1 Return to Fuzzy Sets

For the sake of convenience, usually a fuzzy set is denoted as:

$$A = \mu_A(x_1)/x_1 + \cdots + \mu_A(x_n)/x_n$$

that belongs to a finite universe of discourse:

$$A \subseteq \{x_1, x_2, \ldots, x_n\} = X$$

where $\mu_A(x_i)/x_i$ (a singleton) is a pair "grade of membership element".

3.1.2 Example

Consider $X = \{1, 2, \ldots, 10\}$. Suppose a child is asked which of the numbers in X are "large" relative to the others. The child might come up with the following:

Number	Comment	Degree
10	Definitely	1
9	Definitely	1
8	Quite possible	0.8
7	Maybe	0.5
6	Not usually	0.2
5, 4, 3, 2, 1	Definitely Not	0

From this, we can define some new terms using different membership functions:

Small	$\mu_B = 1 - \mu_A$
Very Large	$\mu_C = (\mu_A)^2$
Large-ish	$\mu_D = \sqrt{\mu_A}$

3.2 Definitions on Fuzzy Sets

We have the following definitions for two fuzzy sets (A, μ_A) and (B, μ_B), where $A, B \subseteq X$:

- Equality: $A = B$ iff $\mu_A(x) = \mu_B(x)$ for all $x \in X$
- Inclusion: $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ for all $x \in X$
- Cardinality: $|A| = \sum_{i=1}^{n} \mu_A(x_i)$
- Empty Set: A is empty iff $\mu_A(x) = 0$ for all $x \in X$.
- α-Cut: Given $\alpha \in [0, 1]$, the α-cut of A is defined by $A_\alpha = \{x \in X \mid \mu_A(x) \geq \alpha\}$

3.3 Operations on Fuzzy Sets

Let $(A, \mu_A), (B, \mu_B)$ be a fuzzy sets.

- Complementation: $(\neg A, \mu_{\neg A})$, where $\mu_{\neg A} = 1 - \mu_A$
- Height: $h(A) = \max_{x \in X} \mu_A(x)$
- Support: $\text{supp}(A) = \{x \in X \mid \mu_A(x) > 0\}$
- Core: $C(A) = \{x \in X \mid \mu_A(x) = 1\}$
- Intersection: $C = A \cap B$, where $\mu_C = \min_{x \in X}\{\mu_A, \mu_B\}$
- Union: $C = A \cup B$, where $\mu_C = \max_{x \in X}\{\mu_A, \mu_B\}$
- Bounded Sum: $C = A + B$, where $\mu_C(x) = \min\{1, \mu_A(x) + \mu_B(x)\}$
- Bounded Difference: $C = A - B$, where $\mu_C(x) = \max\{0, \mu_A(x) - \mu_B(x)\}$

- <u>Exponentiation</u>: $C = A^\alpha$ where $\mu_C = (\mu_A)^\alpha$ for $\alpha > 0$
- <u>Level Set</u>: $C = \alpha A$ where $\mu_C = \alpha\mu_A$ for $\alpha \in [0, 1]$
- <u>Concentration</u>: $C = A^\alpha$ where $\alpha > 1$
- <u>Dilation</u>: $C = A^\alpha$ where $\alpha < 1$

Note that $A \cap \neg A$ is not necessarily the empty set, as would be the case with classical set theory. Also, if A is crisp, then $A^\alpha = A$ for all α. We will define the Cartesian product $A \times B$ to be the same as $A \cap B$.

3.4 Membership Functions

We will usually consider one of the following membership functions:

- <u>Triangular</u>: $\text{tri}(x; a, b, c) = \max\left\{\min\left\{\frac{x-a}{b-a}, \frac{c-x}{c-b}\right\}, 0\right\}$
- <u>Trapezoidal</u>: $\text{trap}(x; a, b, c, d) = \max\left\{\min\left\{\frac{x-a}{b-a}, \frac{d-x}{d-c}, 1\right\}, 0\right\}$
- <u>Gaussian</u>: $\text{gauss}(x; c, \sigma) = \exp\left[-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2\right]$
- <u>Generalised Bell</u>: $\text{gbell}(x; a, b) = \dfrac{1}{1 + \left|\frac{x-b}{a}\right|^{2a}}$

3.5 Fuzzy Relationships

In order to understand how two fuzzy subsets are mapped onto each other to obtain a cross product, consider the example of an air-conditioning system. Air-conditioning involves the delivery of air which can be warmed or cooled and have its humidity raised or lowered.

An air-conditioner is an apparatus for controlling, especially lowering, the temperature and humidity of an enclosed space. An air-conditioner typically has a fan which blows/cools/circulates fresh air and has cooler and the cooler is under thermostatic control. Generally, the amount of air being compressed is proportional to the ambient temperature.

Consider an air-conditioner which has five control switches: cold, cool, pleasant, warm and hot. The corresponding speeds of the motor controlling the fan on the air-conditioner has the graduations: minimal, slow, medium, fast, blast. The rules governing the air-conditioner are as follows:

Rule		Temperature		Speed
1		Cold		Minimal
2		Cool		Slow
3	IF	Pleasant	THEN	Medium
4		Warm		Fast
5		Hot		Blast

Hence, we can regard our rules as the cross product of temperature and speed.

3.6 Rules and Patches

A <u>Fuzzy Patch</u> is defined by a fuzzy rule: a patch is a mapping of two membership functions, it is a product of two geometrical objects, line segments, triangles, squares etc.

In a fuzzy controller, a rule in the rule set of the controller can be visualized as a "device" for generating the product of the input/output fuzzy sets.

Geometrically a patch is an area that represents the causal association between the cause (the inputs) and the effect (the outputs). The size of the patch indicates the vagueness implicit in the rule as expressed through the membership functions of the inputs and outputs.

The total area occupied by a patch is an indication of the vagueness of a given rule that can be used to generate the patch.

Consider a one-input-one output rule: if the input is crisp and the output is fuzzy then the patch becomes a line. And, if both are crisp sets then the patch is vanishingly small - a point.

Chapter 4

Knowledge Representation

Once we have found that the knowledge of a specialism can be expressed through linguistic variables and rules of thumb, that involve imprecise antecedents and consequents, then we have a basis of a knowledge-base. In this knowledge-base "facts" are represented through linguistic variables and the rules follow fuzzy logic. In traditional expert systems facts are stated crisply and rules follow classical propositional logic.

Fuzzification

The problem is that in the "real" world some of our knowledge of facts is derived from the use of sensors: *quantity of heat* measured in degrees Centigrade, *length* measured in meters, *weight* (the quantity of matter) measured in grams, etc. This quantitative, and rather precise factual information has to be mapped onto the term-set of a linguistic variable - the process of FUZZIFICATION.

Inference

Once mapped, the rules within a knowledge base are invoked systematically to see which of the rules is fired and to what degree - the process of INFERENCE. In traditional expert systems, only those rules fire whose antecedents are true. In fuzzy expert systems rules may fire to a certain degree: all rules may fire to a degree between zero and unity. Min and Product are two inference methods.

1. In Min inferencing the output membership function is clipped off at a height corresponding to the computed degree of truth of a rule's premise. This corresponds to the traditional interpretation of the fuzzy logic's AND operation.

2. In Product inferencing the output membership function is scaled by the premise's computed degree of truth.

Composition

But, of course, we have to see what influence each rule has given the fuzzy input values. An "averaging" procedure is adopted to compute the effective contribution of each of the rules. This is the process of COMPOSITION. Max and Sum are two composition rules:

1. In MAX composition, the combined fuzzy subset is constructed by taking the pointwise maximum over all the fuzzy subsets assigned to the output variable by the inference rule.
2. The SUM composition, the combined output fuzzy subset is constructed by taking the pointwise sum over all the fuzzy subsets assigned to output variable by their inference rule. (Note that this can result in truth values greater than 1).

Defuzzification

And, finally, we have to convert the fuzzy values outputted by the inference procedure onto a crisp number that can be used in the "real" world. This process is called DEFUZZIFICATION.

Example: Air Conditioner

We'll return to the above example of an air conditioner. Suppose that we require that the air-conditioner operates at $16°C$. Then our above processes are as follows:

- **Fuzzification**: $16°C$ corresponds to Cool/Pleasant.
- **Inference**: Check the rules which contain the above linguistic variables.
- **Composition**: Create new membership function of the alpha levelled functions for Cool and Pleasant.
- **Defuzzification**: Examine the fuzzy sets of Slow and Medium and obtain a speed value.

4.1 Knowledge-Based Systems

This is a computer program which, with its associated data, embodies organised knowledge concerning some specific area of human activity. Such a system is expected to perform competently, skilfully and in a cost-effective manner. It may be thought of as a computer program which mimics the performance of a human expert.

A FUZZY KNOWLEDGE-BASED SYSTEM (KBS) is a KBS that performs approximate reasoning. Typically a fuzzy KBS uses knowledge representation and reasoning in systems that are based on the application of Fuzzy Set Theory. A fuzzy knowledge base comprises vague facts and vague rules of the form:

KB Entity	Fuzzy KB	Crisp KB
Fact	X is μ_X	X is true or X is not true
Rule	If X is μ_X, then Y is μ_Y	If X then Y

There are two challenges of this:

1. How to interpret and how to represent vague rules with the help of appropriate fuzzy sets?

2. How to find an inference mechanism that is founded on well-defined semantics and that permits approximate reasoning by means of a conjunctive general system of vague rules and case-specific vague facts?

The fuzzy rules are also called LINGUISTIC RULES comprising an ANTECEDENT/PREMISE (the IF part) and a CONSEQUENT/CONCLUSION (the THEN part). The antecedent/premise describes an object or event or state in the form of a fuzzy specification of a measured value. The consequent/conclusion specifies an appropriate fuzzy value.

4.2 Linguistic Variable

Informally, a LINGUISTIC VARIABLE is a variable whose values are words or sentences in a natural or artificial language. For example, if **age** is interpreted as a linguistic variable, then its term-set $T(\cdot)$, (that is, the set of its linguistic values) might be

$$T(\text{age}) = \text{young} + \text{old} + \text{very young} + \text{not young} + \text{very old} + \text{very very young} + \ \dots$$

where each of the terms in $T(\text{age})$ is a label of a fuzzy subset of a universe of discourse, say $U = [0, 100]$. A linguistic variable is associated with two rules:

1. a SYNTACTIC RULE, which defines the well-formed sentences in $T(\cdot)$

2. a SEMANTIC RULE, by which the meaning of the terms in T( ) may be determined. If X is a term in $T(\cdot)$, then its meaning (in a denotational sense) is a subset of U. A primary fuzzy set, that is, a term whose meaning must be defined a priori, and serves as a basis for the computation of the meaning of the nonprimary terms in $T(\cdot)$.

Example 4.2.1 (Age)**.** The primary terms in the equation above are **young** and **old**, whose meaning might be defined by their respective compatibility functions μ_Y and μ_O. From these, then, the meaning - or, equivalently, the compatibility functions - of the non-primary terms in Tage) above may be computed by the application of a semantic rule.

$$\mu_{\text{very young}} = (\mu_Y)^2$$
$$\mu_{\text{more or less old}} = \sqrt{\mu_O}$$
$$\mu_{\text{not very young}} = 1 - (\mu_Y)^2$$

Example 4.2.2 (Air Conditioner)**.** Consider again the problem of controlling an air-conditioner. The rules that are used to control the air conditioner can be expressed as a cross product:

$$\text{Control} = \text{Temperature} \times \text{Speed}$$

Where the set of linguistic values of the term sets is given as

$$\text{Temperature} = \text{Cold} + \text{Cool} + \text{Pleasant} + \text{Warm} + \text{Hot}$$

$$\text{Speed} = \text{Minimal} + \text{Slow} + \text{Medium} + \text{Fast} + \text{Blast}$$

Recalling the rules, we could define our membership functions for Speed as:

Term	Membership Function
Minimal	$\mu_{\text{Minimal}}(V) = \min\left(\dfrac{1-V}{30}, 1\right)$
Slow	$\mu_{\text{Slow}}(V) = \max\left(\min\left(\dfrac{V-10}{20}, \dfrac{50-V}{20}\right), 0\right)$
Medium	$\mu_{\text{Medium}}(V) = \max\left(\min\left(\frac{V-40}{10}, \dfrac{60-V}{10}\right), 0\right)$
Fast	$\mu_{\text{Fast}}(V) = \max\left(\min\left(\dfrac{V-50}{20}, \dfrac{90-V}{20}\right), 0\right)$
Blast	$\mu_{\text{Blast}}(V) = \min\left(\dfrac{V-70}{30}, 1\right)$

Similarly, we could define our membership functions for Temperature as:

Term	Membership Function
Cold	$\mu_{\text{Cold}}(T) = \min\left(\dfrac{10-T}{10}, 1\right)$
Cool	$\mu_{\text{Cool}}(T) = \max\left(\min\left(\dfrac{T}{12.5}, \dfrac{17.5-V}{5}\right), 0\right)$
Pleasant	$\mu_{\text{Pleasant}}(T) = \max\left(\min\left(\dfrac{T-15}{7.5}, -\dfrac{20-T}{2.5}\right), 0\right)$
Warm	$\mu_{\text{Warm}}(T) = \max\left(\min\left(\dfrac{T-17.5}{5}, \dfrac{27.5-T}{5}\right), 0\right)$
Hot	$\mu_{\text{Hot}}(T) = \min\left(\dfrac{V-25}{5}, 1\right)$

Now, for our fuzzification, we consider that the temperature is $16°C$ and we want our knowledge base to compute the speed. The fuzzification of the the crisp temperature gives the following membership for the Temperature fuzzy set:

	μ_{Cold}	μ_{Cool}	μ_{Pleasant}	μ_{Warm}	μ_{Hot}
$T = 16$	0	0.3	0.4	0	0
Rule	1	2	3	4	5
Does it fire?	No	No	Yes	Yes	No

Inference: Consider that the temperature is 16° and we want our knowledge base to compute the speed. Rules 2 and 3 are firing and are essentially the fuzzy patches made out of the cross products of

$$\text{Cool} \times \text{Slow}$$

$$\text{Pleasant} \times \text{Medium}$$

Composition: The Cool and Pleasant sets have an output of 0.3 and 0.4 respectively. The fuzzy sets for Slow and Medium have to be given an α-level cut for these output values respectively.

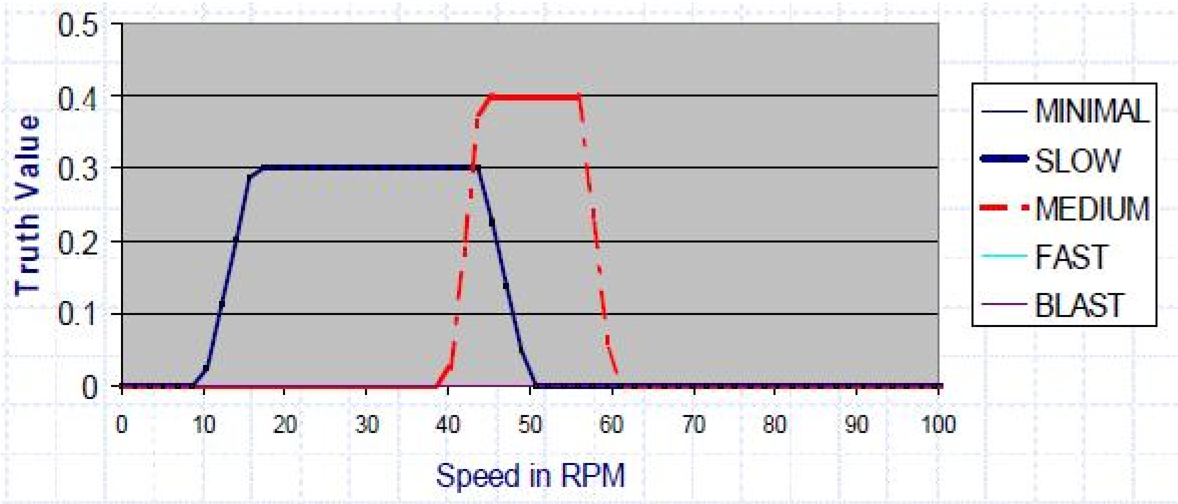

Table 4.1: α-levelled Fuzzy Sets for Speed

The fuzzy output from rules 2 and 3 can be given by Defuzzification: Now we have to find a way to obtain one single number from the curve below - one number corresponding to the speed of the air-conditioner's motor.

4.3 Defuzzification Techniques

Two popular defuzzification techniques are the Centroid and Maximum Method.

4.3.1 Centroid

We can compute the 'Centre of Gravity (COG) or 'Centre of Area' (COA) of the output of the rules: The COG involves the computation of the weighted sum of the Speed and the corresponding membership function of the output fuzzy set and the weighted sum of the membership function.

$$\eta = \frac{1}{\displaystyle\int_Y \mu_{x_1,\ldots,x_n}(y)\,dy} \int_Y y\mu_{x_1,\ldots,x_n}(y)\,dy$$

The centre of gravity approach attempts to take the rules into consideration according to their degree of applicability. If a rule dominates during a certain interval then its dominance is discounted in other intervals. There are problems with the notion of COG and COA. The crisp value η can be obtained by approximating the integral with a sum

$$\eta = \frac{1}{\displaystyle\sum_{y\in Y} \mu_{x_1,\ldots,x_n}(y)} \sum_{y\in Y} y\mu_{x_1,\ldots,x_n}(y)$$

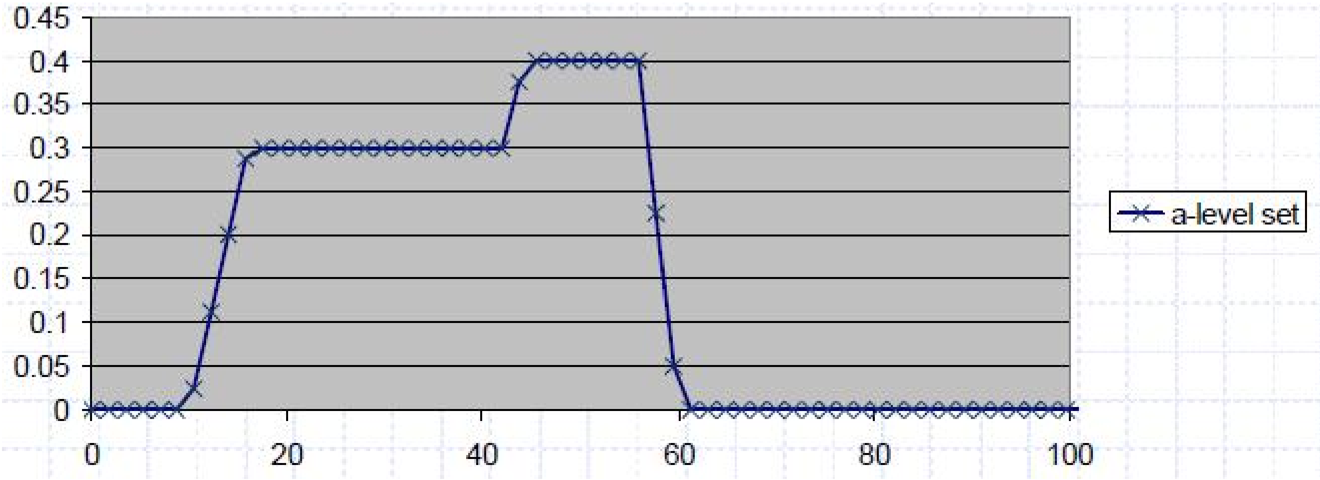

Table 4.2: Rules

The centre of gravity approach attempts to take the rules into consideration according to their degree of applicability. If a rule dominates during a certain interval then its dominance is discounted in other intervals.

4.3.2 Maximum

Here again the weighted sum and weighted membership are worked out, except that the membership function is given another alpha level cut corresponding to the maximum value of the output fuzzy set. The crisp value for MOM method is given as:

$$\eta = ***$$

Note

Both of these methods give different results:

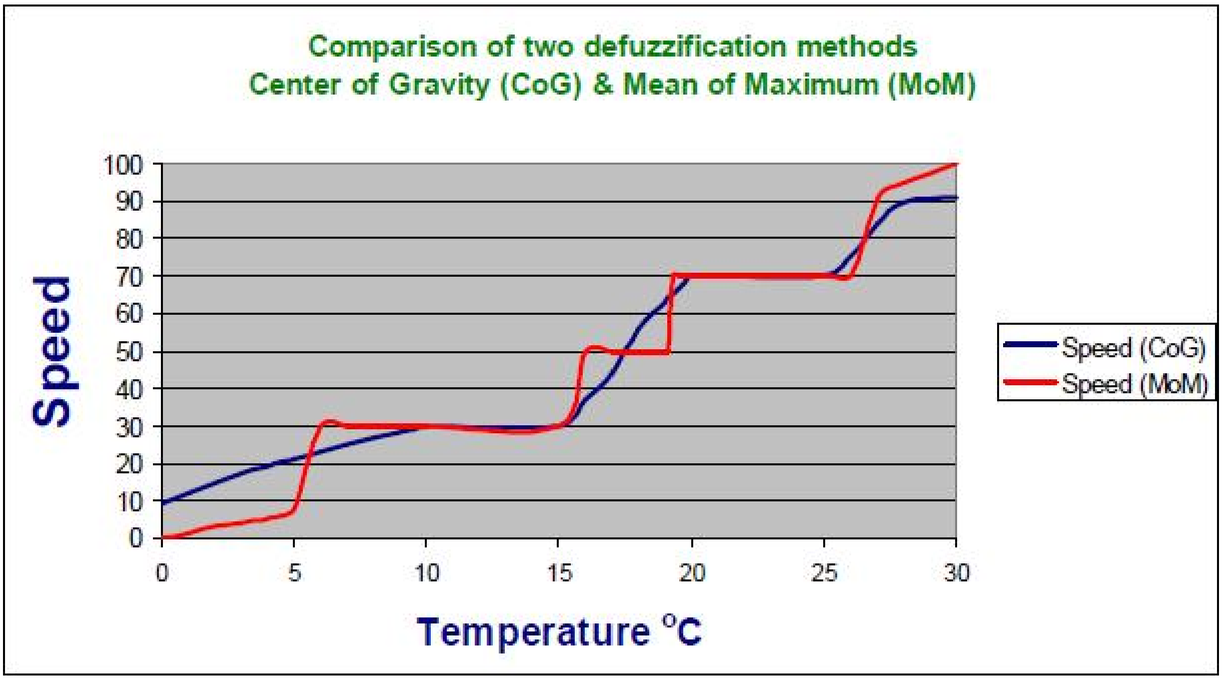

Table 4.3: CoG method versus MoM method

Chapter 5

Fuzzy Control

The term control is generally defined as a mechanism used to guide or regulate the operation of a machine, apparatus or constellations of machines and apparatus. Often the notion of control is inextricably linked with feedback: a process of returning to the input of a device a fraction of the output signal. Feedback can be negative, whereby feedback opposes and therefore reduces the input, and feedback can be positive whereby feedback reinforces the input signal.

'Feedback control' is thus a mechanism for guiding or regulating the operation of a system or subsystems by returning to the input of the (sub)system a fraction of the output. The machinery or apparatus etc., to be guided or regulated is denoted by S, the input by W and the output by y, and the feedback controller by C. The input to the controller is the so-called error signal e and the purpose of the controller is to guarantee a desired response of the output y.

One can intuitively argue that the control signal, u, in part, is

- Proportional to the error;
- Proportional to both the magnitude of the error and the duration of the error
- Proportional to the relative changes in the error values over time

5.1 Conventional Control

In the case of classical operations of process control one has to solve the non-linear function u. Furthermore, it is very important that one also finds the proportionality constants K_I, K_D, and K_P. In the case of fuzzy controller, the non-linear function is represented by a fuzzy mapping, typically acquired from human beings. The above intuition can be expressed more formally as an algebraic equation involving three proportionality constants:

$$u(t) = K_P e(t) + K_I \int_0^t e(\tau)\, d\tau + K_D \frac{de}{dt}$$

Value	determines reaction to the
Proportional (K_P)	current error
Integral (K_I)	sum of recent errors
Derivative (K_D)	rate at which the error has been changing

The weighted sum of these three actions is used to adjust the process via a control element such as the position of a control valve or the power supply of a heating element. Conventional control theory uses a mathematical model of a process to be controlled and specifications of the desired closed-loop behaviour to design a controller. This approach may fall short if the model of the process:

- is difficult to obtain, or
- is (partly) unknown, or
- is highly non-linear.

5.2 Fuzzy Controller

Logical rules with vague predicates can be used to derive inference from vague formulated data. The idea of linguistic control algorithms was a brilliant generalisation of the human experience to use linguistic rules with vague predicates in order to formulate control actions. A FUZZY CONTROLLER is a device that is intended to modelize some vaguely known or vaguely described process.

A knowledge-based system for closed-loop control is a control system which enhances the performance, reliability, and robustness of control by incorporating knowledge which cannot be accommodated in the analytic model upon which the design of a control algorithm is based, and that is usually taken care of manual modes of operation, or by other safety and ancillary logic mechanisms.

There are two types of fuzzy controllers:

Controller Type	Typical Operation
Mamdani (linguistic) Controller	Direct closed-loop controller
Takagi-Sugeno-Kang Controller	Supervisory controller

The controller can be used with the process in two modes: *Feedback* mode when the fuzzy controller will act as a control device, and *Feed Forward* mode where the controller can be used as a prediction device. All inputs to, and outputs from, the controller are in the form of linguistic variables. In many ways, a fuzzy controller maps the input variables into a set of output linguistic variables. Usually, a plant, process, vehicle, or any other object to be controlled is called a system S. The feedback controller is expected to 'guarantee a desired response', or output y.

Regulation is a process described in the control theory literature as a process for 'keeping the output y close to the setpoint (reference input) w, despite the presence of disturbances, fluctuations of the system parameters, and noise measurements'. (Error $e = w - y$). A controller is implemented using the control algorithm.