The Zeeman Effect

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Abstract

The purpose of this experiment was to study the splitting of a spectral line due to a magnetic field, known as the Zeeman effect. Specifically the cadmium red line was investigated. There were three main parts to this:

- The wavelength of the cadmium red line, the light source used in this experiment, was calculated. This was found to be 660nm ± 20nm, which agrees within experimental error of the accepted value of 643.84nm.

- The Transverse Zeeman effect was investigated and it was shown that the shift in wavelength of the outer triplet lines, $\Delta\lambda_0$, is proportional to the applied magnetic field, B. The Bohr magneton was found to be $9.5 \pm 0.5 \times 10^{-24} JT^{-1}$ which agrees within experimental error of the accepted value of $9.274 \times 10^{-24} JT^{-1}$. In addition, the polarisation state of the outer triplet lines compared to the inner line was found to be 90°.

- The Longitudinal Zeeman effect was investigated and it was shown that there is the same shift $\Delta\lambda_0$ in the doublet lines as there is in the transverse configuration. Finally, the doublet lines were found to be circularly polarised and at 90° to each other.

Introduction and Basic Theory

The Zeeman Effect is the splitting of a spectral line in the presence of a static magnetic field. This is due to the magnetic field breaking the degeneracy of electron configurations with the same energy. It was first discovered by the Dutch physicist Pieter Zeeman in 1896. The interaction between atoms and field is classified depending on the strength of the field.

- For strong fields the Paschen-Back effect is used.

- For weak fields the Zeeman effect is used.

Within the Zeeman effect itself there are also two seperate categories; the normal Zeeman effect and the anamolous Zeeman effect. The normal Zeeman effect agrees with the classical theory of Lorentz. It describes transitions between states that both have $S = 0$. The anomalous effect depends on electron spin, and is purely quantum mechanical. It describes transitions between states that both have $S \neq 0$.

The total angular momentum of an electron is the sum of spin and orbital contributions:

$$\hat{J} = \hat{L} + \hat{S}$$

The spectroscopic state of an atom is defined by the eigenvalues of J, S and L, and can be written as ${}^{2S+1}L_J$, where S, P, D, F are used for values of L.

The interaction term, V_M, is the perturbation due to the magnetic field:

$$V_M = -\vec{\mu} \cdot \vec{B}$$

Where μ is the magnetic moment of the atom:

$$\vec{\mu} = -\mu_B \vec{J} g / \hbar$$

If V_M is small (less than the fine structure) then this is the proper Zeeman effect. The Paschen-Back effect is when V_M exceeds the LS coupling significantly. μ_B is known as the Bohr magneton and g is the Lande g value, given by:

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

For example, the $2J+1$ levels with different M_J have the same energy when $B = 0$. However they are split when a magnetic field is applied. To predict the spectrum, the allowed transitions between various energy levels must be known. In additon, the probability of each transition determines the intensity of each line. These rules are included in the appendix.

For the the transverse normal Zeeman effect in cadmium, there will be 3 lines at wavelengths λ_0, $\lambda_0 \pm \Delta\lambda$ and 2 lines at $\lambda_0 \pm \Delta\lambda$ for the longitudinal normal effect where:

$$\Delta\lambda = \lambda_0^2 \mu_B B / (hc) \tag{1}$$

Experimental Setup

The equipment used in this experiment was as follows:

- A cadmium light source
- An elecromagnet
- Two lenses, L1 and L2
- A Fabry Perot etalon
- A filter
- An eyepiece and a CCD connected to a computer

The filter is used so that only red light is permitted. This stops the anamalous Zeeman effect of the blue light interfering with the results. The etalon is used to split a single incidence ray into muiltiple rays. In conjuction with the lenses, this produces mutliple beam interference with sharp ring shaped fringes. The strength and orientation of the magnetic field can be varied using the electromagnet. In additon, a linear polariser and a quarter wave plate were used to investigate the polarisation state of the lines.

The wavelength of the light was found using the following:

$$m\lambda_0 = 2nd[1 - (r_m^2 / 2n^2 f^2)] \tag{2}$$

Where r_m is the radius of ring of order number m, n is the refractive index (1.457), d is the etalon thickness ($4.0mm$) and f is the focal length of the lens ($150mm$). As such, to a good approximation:

$$\Delta\lambda/\lambda_0 = -r\Delta r/(n^2 f^2) \tag{3}$$

Where Δr is the change in the ring radius, r, due to the change in λ of $\Delta\lambda$. Note that δ will denote errors from now on to avoid confusion.

1. Wavelength of the cadmium red line

The aim of this section was to determine the wavelength, λ_0, of the cadmium red line when there is no magnetic field acting on it.
The lenses L1 and L2 were adjusted as so that a sufficiently sharp image of the rings was observed. Equation (2) shows that λ_0 can be found if ${r_m}^2$ is plotted versus m, as the slope will be equal to $(\lambda_0 n f^2/d)$. As such, values for r_m for the first 10 rings were found using the CCD and computer software. Note that one pixel is equivalent to $14\mu m$.

2. Transverse normal Zeeman effect

The aims for this section were to:

- determine the shift $\pm\Delta\lambda$ of the outer triplet lines and show that $\Delta\lambda \propto B$.
- determine the value of the Bohr magneton μ_B.
- determine the polarisation state of each of the triplet lines.

Equation (3) shows that $\Delta\lambda$ can be found by measuring r and Δr, and then μ_B can be found using equation (1). For this, the magnetic field was increased by increasing the current, I. Graphs were taken for values of I of 4A, 5A, 6A and 7A. Then values of r, Δr and $r\Delta r$ were tabluated and compared. A graph of the mean value of $r\Delta r$ vs B was plotted. The Bohr magneton, μ_B, was also calculated. Finally, the polarisation state of the lines was determined using the linear polariser and quarter wave plate.

3. Longitudinal normal Zeeman effect

The aims for this section were to:

- to show that there is the same shift $\pm\Delta\lambda$ in λ, for a given B, as the transverse configuration.
- determine the polarisation state of the two doublet lines.

For this, the magnet was roted through 90°, and the experiment was repeated as above.

Results

1. Wavelength of the cadmium red line

The recorded spectrum seen is shown in Figure 1. Using this, the results found were:

Peak No.	Pixel Gap	${r_m}^2 (m^2)$	$\delta {r_m}^2 (m^2)$
1	0	0	0
2	333	5.43×10^{-6}	7×10^{-7}
3	470	1.08×10^{-5}	9×10^{-7}
4	576	1.62×10^{-5}	1×10^{-6}
5	664	2.16×10^{-5}	1×10^{-6}
6	744	2.71×10^{-5}	1×10^{-6}
7	813	3.23×10^{-5}	2×10^{-6}
8	878	3.77×10^{-5}	2×10^{-6}
9	938	4.314×10^{-5}	2×10^{-6}
10	995	4.85×10^{-5}	2×10^{-6}

A plot of r_m^2 vs m was plotted and is shown in Figure 2. From the slope, M for clarity, the wavelength λ_0 was calculated as follows:

	Value	Error (±)
M	$5.39\times10^{-6} m^2$	$6\times10^{-9} m^2$
d	4.0×10^{-3}m	1×10^{-4}m
n	1.457	0.001
f^2	$2.25\times10^{-2} m^2$	$2\times10^{-4} m^2$
λ_0	6.58×10^{-7}m	2×10^{-8}m

This value of $\lambda_0 = 660 \pm 20$nm agrees within experimental error ($\sim 5\%$) of the accepted value of 643.84nm.

2. Transverse normal Zeeman effect

While the magnetic field of the electromagnet varies with current, it is not completely linear. B can have slightly different values depending on whether the current was increasing or decreasing at the time. This is known as a hysteresis loop. The values of B at varying I were given as follows:

I(A)	B(T) for increasing I	B(T) for decreasing I
4	0.394	0.407
5	0.485	0.493
6	0.552	0.555
7	0.594	0.594

The different spectra for 4A, 5A, 6A and 7A were recorded and are seen in Figure 3. From this, the values of r, Δr and rΔr were found for different currents:

$I = 4A$

r(m)	δr(m)	Δr(m)	$\delta\Delta$r(m)	rΔr(m^2)	δrΔr(m^2)
0.001561	0.000014	0.0003745	0.000014	5.85×10^{-7}	3×10^{-8}
0.002807	0.000014	0.000203	0.000014	5.69×10^{-7}	4×10^{-8}
0.003633	0.000014	0.000154	0.000014	5.59×10^{-7}	5×10^{-8}
0.004312	0.000014	0.000126	0.000014	5.43×10^{-7}	6×10^{-8}
0.004886	0.000014	0.0001015	0.000014	4.96×10^{-7}	7×10^{-8}

$I = 5A$

r(m)	δr(m)	Δr(m)	$\delta\Delta$r(m)	rΔr(m^2)	δrΔr(m^2)
0.001568	0.000014	0.00049	0.000014	7.68×10^{-7}	3×10^{-8}
0.002807	0.000014	0.0002555	0.000014	7.17×10^{-7}	4×10^{-8}
0.003633	0.000014	0.000196	0.000014	7.12×10^{-7}	5×10^{-8}
0.004305	0.000014	0.0001645	0.000014	7.08×10^{-7}	6×10^{-8}
0.004886	0.000014	0.000147	0.000014	7.18×10^{-7}	7×10^{-8}
0.005404	0.000014	0.000133	0.000014	7.19×10^{-7}	8×10^{-8}

$I = 6A$

r(m)	δr(m)	Δr(m)	$\delta\Delta$r(m)	rΔr(m^2)	δrΔr(m^2)
0.001568	0.000014	0.0005775	0.000014	9.06×10^{-7}	3×10^{-8}
0.002807	0.000014	0.0002975	0.000014	8.35×10^{-7}	4×10^{-8}
0.003633	0.000014	0.000224	0.000014	8.14×10^{-7}	5×10^{-8}
0.004312	0.000014	0.0001925	0.000014	8.30×10^{-7}	6×10^{-8}
0.004886	0.000014	0.0001645	0.000014	8.04×10^{-7}	7×10^{-8}
0.005404	0.000014	0.0001505	0.000014	8.13×10^{-7}	8×10^{-8}

$I = 7A$

r(m)	δr(m)	Δr(m)	$\delta\Delta$r(m)	rΔr(m^2)	δrΔr(m^2)
0.001568	0.000014	0.000637	0.000014	9.99×10^{-7}	3×10^{-8}
0.002807	0.000014	0.000322	0.000014	9.04×10^{-7}	4×10^{-8}
0.003633	0.000014	0.0002485	0.000014	9.03×10^{-7}	5×10^{-8}
0.004305	0.000014	0.0002065	0.000014	8.89×10^{-7}	$6.\times10^{-8}$
0.004886	0.000014	0.0001855	0.000014	9.06×10^{-7}	7×10^{-8}
0.005404	0.000014	0.0001645	0.000014	8.89×10^{-7}	8×10^{-8}

The average value of rΔr was plotted against B for each different I in Figure 4. The linear trend is clearly visible which shows that $r\Delta r \propto B$ and therefore from equation (3) $\Delta\lambda \propto B$.

Using the slope of the graph and equation (1), the value of the Bohr Magneton, μ_B, was found to be $9.5 \pm 0.5 \times 10^{-24} JT^{-1}$. This also agrees within $\sim 5\%$ experimental error of the accepted value of $9.274 \times 10^{-24} JT^{-1}$

Finally, the polarisation of the lines was found. The linear polariser was used as the quarter wave plate was observed to have no effect. When the polariser was set at 65° the three lines had the same intensity. When rotated to 20°, only the two outer rings were visible. When rotated to 110° only the middle ring was visible. Therefore the outer rings were polarised by 90° ($\frac{\pi}{2}$ rad) with respect to the main (inner) ring.

3. Longitudinal normal Zeeman effect

Similar to above, the spectrum is seen in Figure 5. From this, the values of r, Δr and rΔr were found for 5A and 7A.

$I = 5A$

r(m)	δr(m)	Δr(m)	δΔr(m)	rΔr(m^2)	δrΔr(m^2)
0.0013685	0.000014	0.0005635	0.000014	7.71×10^{-7}	3×10^{-8}
0.0027615	0.000014	0.0002765	0.000014	7.64×10^{-7}	4×10^{-8}
0.003619	0.000014	0.00021	0.000014	7.60×10^{-7}	5×10^{-8}
0.0043155	0.000014	0.0001785	0.000014	7.70×10^{-7}	6×10^{-8}
0.0049105	0.000014	0.0001575	0.000014	7.73×10^{-7}	7×10^{-8}
0.005432	0.000014	0.00014	0.000014	7.60×10^{-7}	8×10^{-8}

$I = 7A$

r(m)	δr(m)	Δr(m)	δΔr(m)	rΔr(m^2)	δrΔr(m^2)
0.0012845	0.000014	0.0007245	0.000014	9.31×10^{-7}	3×10^{-8}
0.002751	0.000014	0.000343	0.000014	9.44×10^{-7}	4×10^{-8}
0.0036225	0.000014	0.0002555	0.000014	9.26×10^{-7}	5×10^{-8}
0.0043155	0.000014	0.0002205	0.000014	9.52×10^{-7}	6×10^{-8}
0.004907	0.000014	0.000189	0.000014	9.27×10^{-7}	7×10^{-8}
0.0054355	0.000014	0.0001715	0.000014	9.32×10^{-7}	8×10^{-8}

The average value of the shift $\Delta\lambda$ in λ_0 for the doublet compared to the triplet is as follows:

5A	Triplet	Doublet
$\Delta\lambda$	1.13×10^{-11}m	1.05×10^{-11}m
$\delta\Delta\lambda$	8×10^{-13}m	8×10^{-13}m

7A	Triplet	Doublet
$\Delta\lambda$	1.28×10^{-11}m	1.29×10^{-11}m
$\delta\Delta\lambda$	8×10^{-13}m	8×10^{-13}m

It is evident that $\Delta\lambda$ is the same for both the longitudinal and transverse Zeeman effect in the same magnetic field.

For the polarisation of the doublet lines, the linear polariser had no effect. However, when used in conjunction with the quarter wave plate (set at 0°), it was observed that both rings would have equal intensity at 15°. The outer ring had a minimum intensity at 65°. Similarly the inner ring had a minimum intensity at −25°. Therfore the rings were circularly polarised and also at 90° ($\frac{\pi}{2}$) to each other.

Conclusion

The aims of this experiment were all fulfilled. Both the longitudinal and the transverse normal Zeeman effects were observed and the shift in wavelength was seen to be proportional to the magnetic field. The wavelength of the cadmium red line was measured as 660nm±20nm. The Bohr magneton was measured as $9.5 \pm 0.5 \times 10^{-24} JT^{-1}$. The polarisation states of the doublet and triplet lines were also found.

Appendix

Experimental Data

Experiment 1

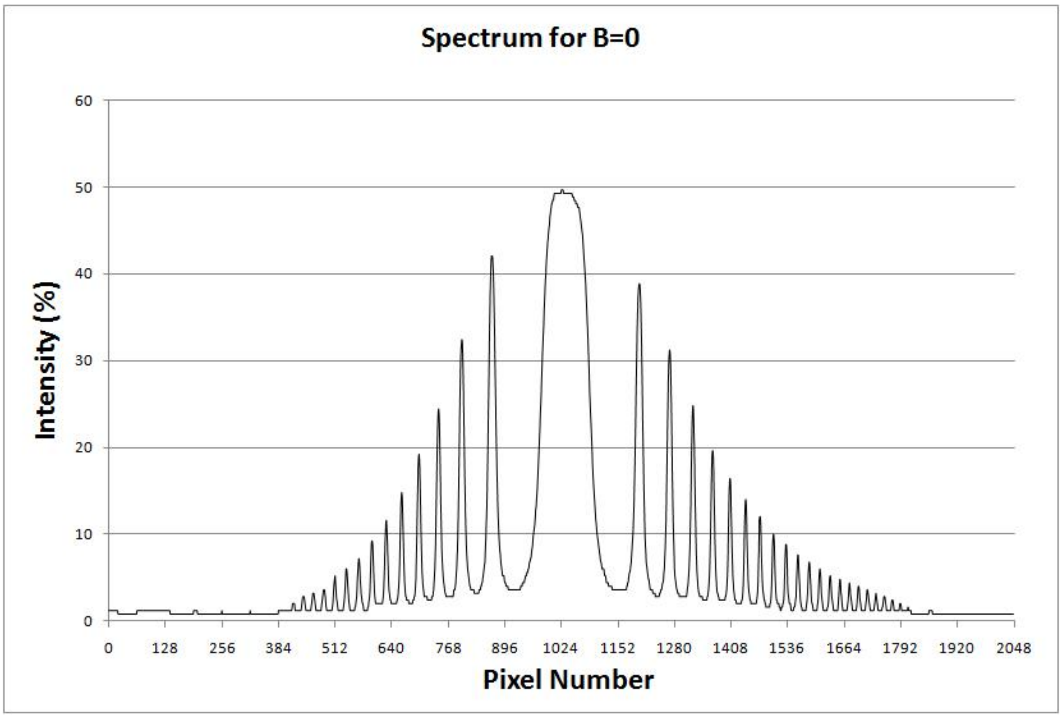

Figure 1: Spectrum for B=0

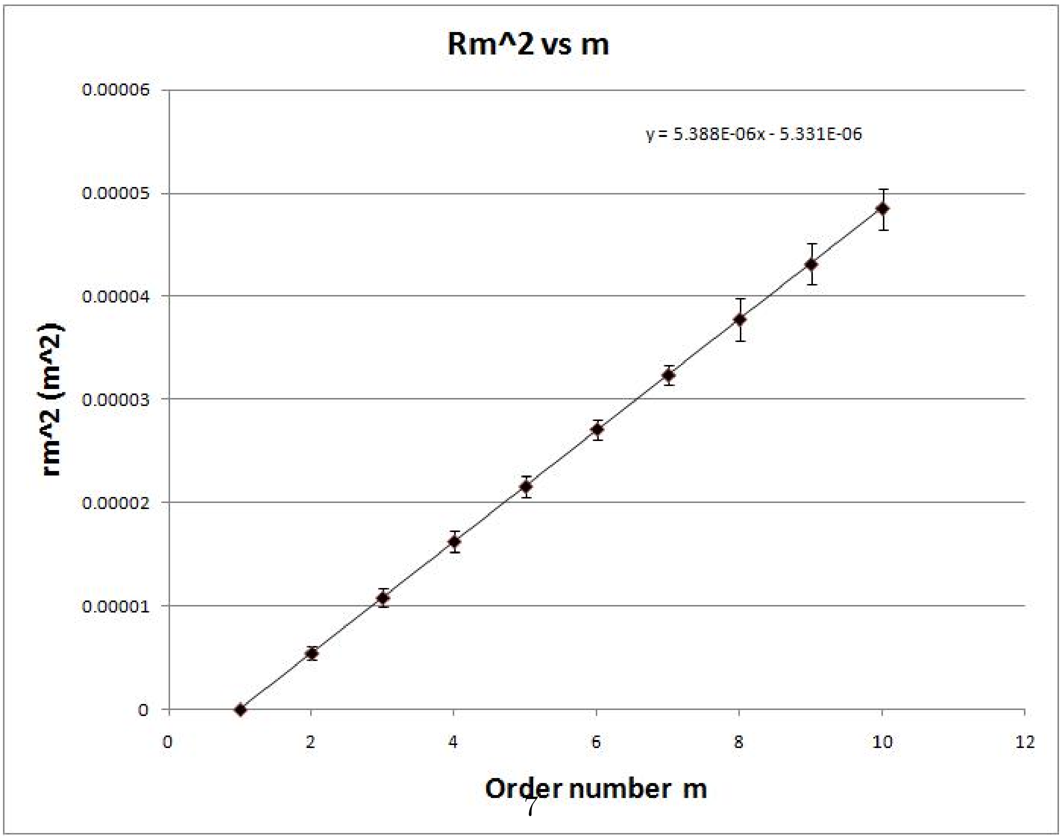

Figure 2: r_m^2 vs m

Experiment 2

The following all show Intensity (%) vs Pixel Number for varying current:

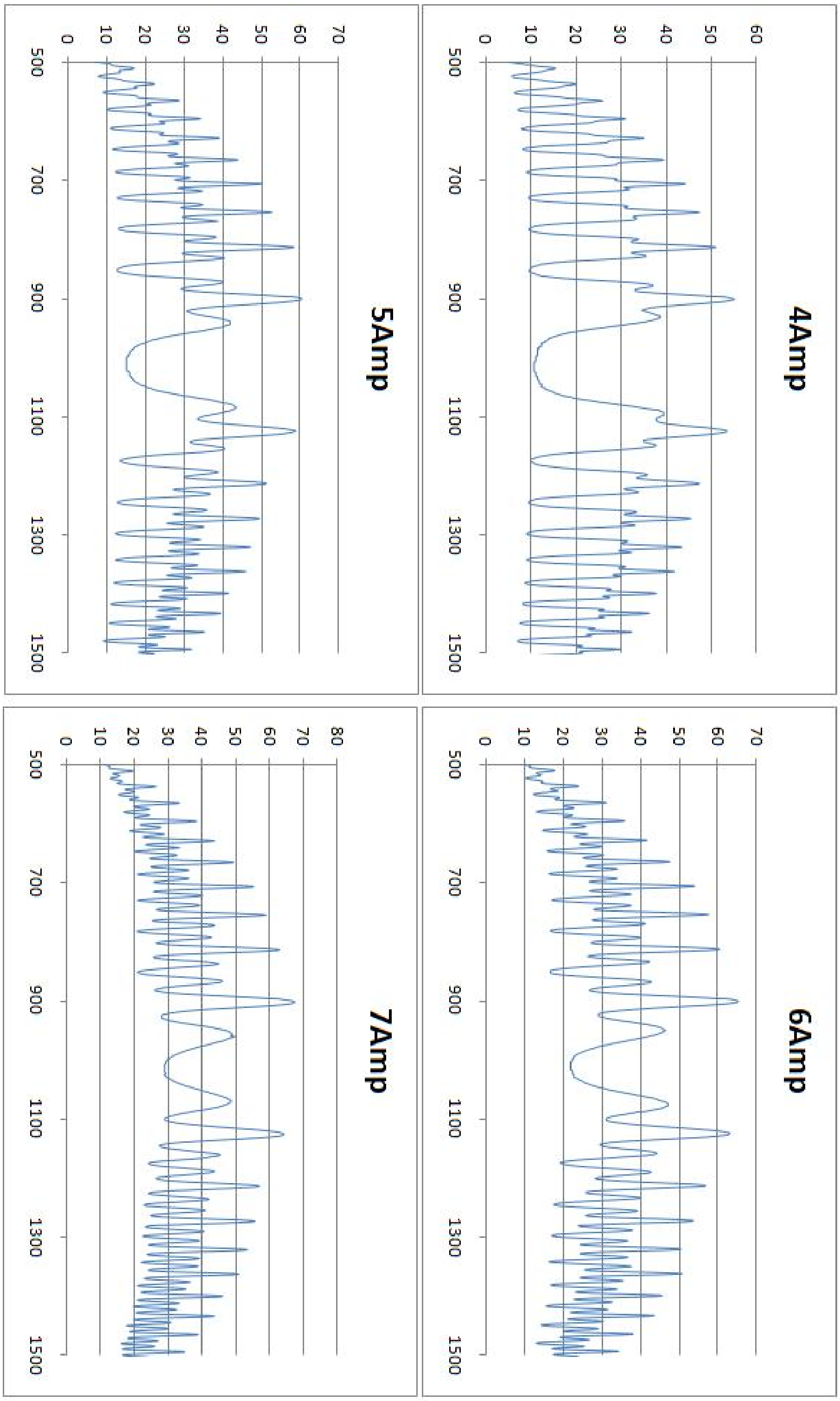

Figure 3: Spectra for various currents

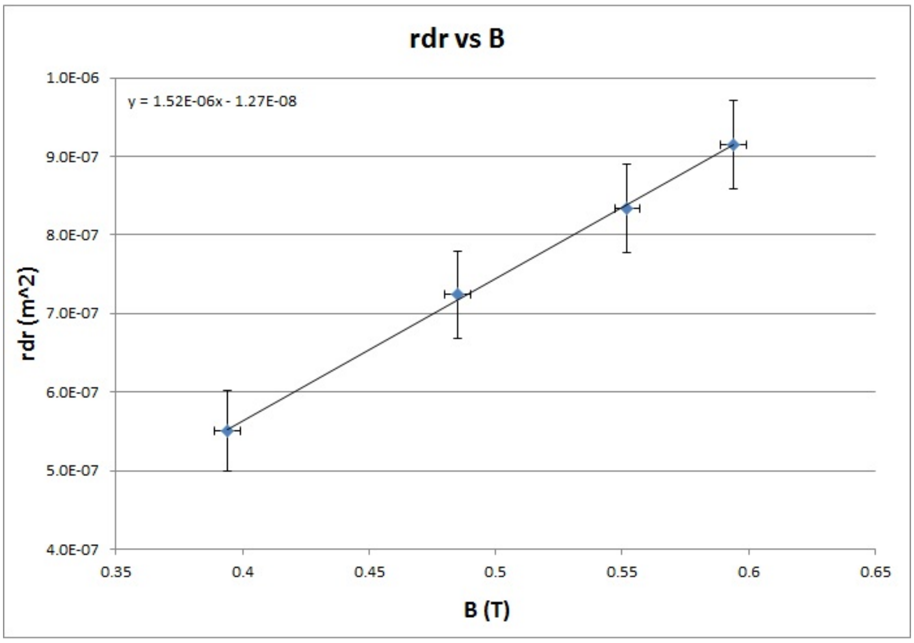

Figure 4: rΔr vs B

Experiment 3

The following all show Intensity (%) vs Pixel Number for varying current:

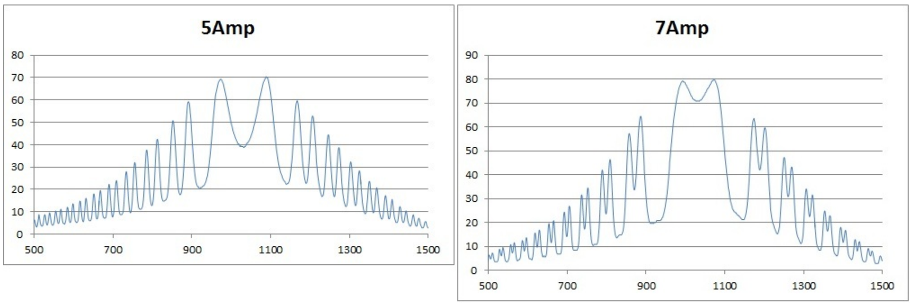

Figure 5: Spectra for various currents

Selection Rules

Selection rules for discrete transitions

		Electric dipole (E1) ("allowed")	**Magnetic dipole (M1) ("forbidden")**	**Electric quadrupole (E2) ("forbidden")**
Rigorous rules	1.	$\Delta J = 0, \pm 1$ (except $0 \not\leftrightarrow 0$)	$\Delta J = 0, \pm 1$ (except $0 \not\leftrightarrow 0$)	$\Delta J = 0, \pm 1, \pm 2$ (except $0 \not\leftrightarrow 0$, $1/2 \not\leftrightarrow 1/2$, $0 \not\leftrightarrow 1$)
	2.	$\Delta M = 0, \pm 1$ (except $0 \not\leftrightarrow 0$ when $\Delta J = 0$)	$\Delta M = 0, \pm 1$ (except $0 \not\leftrightarrow 0$ when $\Delta J = 0$)	$\Delta M = 0, \pm 1, \pm 2$
	3.	Parity change	No parity change	No parity change
With negligible configuration interaction	4.	One electron jumping, with $\Delta l = \pm 1$, Δn arbitrary	No change in electron configuration; i.e., for all electrons, $\Delta l = 0$, $\Delta n = 0$	*No* change in electron configuration; *or* one electron jumping with $\Delta l = 0, \pm 2$, Δn arbitrary
For *LS* coupling only	5.	$\Delta S = 0$	$\Delta S = 0$	$\Delta S = 0$
	6.	$\Delta L = 0, \pm 1$ (except $0 \not\leftrightarrow 0$)	$\Delta L = 0$ $\Delta J = \pm 1$	$\Delta L = 0, \pm 1, \pm 2$ (except $0 \not\leftrightarrow 0$, $0 \not\leftrightarrow 1$)

Errors

The errors were calculated the standard way, $\frac{\Delta A}{A} = \frac{\Delta B}{B} + \frac{\Delta C}{C}$ with the exception of the slopes in the graphs. These were calculated using the excel regression tools.

Sources

Transition selection rules: `http://physics.nist.gov/Pubs/AtSpec/node17.html`

On the Influence of Magnetism on the Nature of the Light Emitted by a Substance, P. Zeeman found at: `http://adsabs.harvard.edu/cgi-bin/nph-data_query?bibcode=1897ApJ.....5..332Z&link_type=GIF`