MA U34605 Quiz 05 w/e 11/12/20

Answer any 3 questions. Submit them using the `submit-work` program as pdfs, either handwritten and scanned, or typeset. They should be submitted before midnight on Thursday 31 December. All questions carry 20 marks.

(1) Here is an (inefficient) procedure to sort an array of strings. Is it stable or unstable? Give reasons.

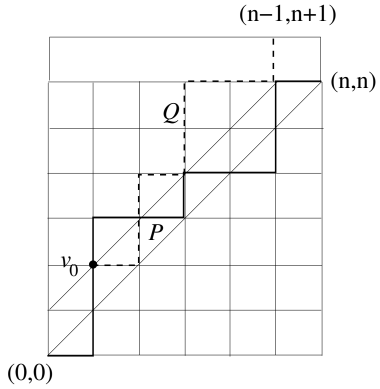
```
void insert ( int k, char * w[], char * s )
{
  int i;
  for (i=k; i>0 && strcmp(w[i-1],s) > 0; --i)
    w[i] = w[i-1];
  w[i] = s;
}

void sort ( int n, char * source[], char * target[] )
{
  int i;
  for (i=0; i<n; ++i)
    insert ( i, target, source[i] );
}
```

(2) Show how a 'naïve' implementation of Dijkstra's algorithm has runtime $O(m+n^2)$ with n vertices and m edges.

(3) Dijkstra's algorithm can be improved, using some extra structure, so that after completion, for every vertex u, it is possible to produce efficiently a shortest path from s to u. Show how this can be done.

(4) There is an ingenious way to calculate b_n, the number of binary trees with n nodes. Let $G_{k,\ell}$ be the *grid graph* with vertices $\{(i,j): \ 0 \le i \le k, 0 \le j \le \ell\}$ and horizontal and vertical edges of unit length. We consider paths in $G_{n,n}$. A path from $(0,0)$ to (n,n) is *correct* if it never goes above the diagonal $i=j$, that is, for every vertex on the path, $i \ge j$. The number of correct paths is b_n: this can be shown. The trick is to show that the number of *incorrect* paths in $G_{n,n}$ equals the number of paths, unconstrained, from $(0,0)$ to $(n-1,n+1)$ in $G_{n-1,n+1}$. The trick is: let D be the above-diagonal $\{(x,y): \ y = x+1\}$. Every incorrect path P must have a vertex on D. Let v_0 be the earliest vertex on the path P which belongs also to D. Let P' be the initial part of the path leading to v_0, P'' the remainder of P, Q' the result of *reflecting* P'' in D, and Q the combination of P' followed by Q'. Then Q is a path from $(0,0)$ to $(n-1,n+1)$ in $G_{n-1,n+1}$. Use these facts to re-calculate b_n.



(n−1,n+1)
(n,n)
Q
P
v_0
(0,0)