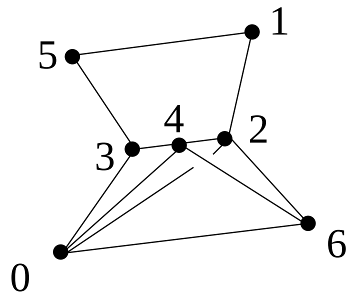


Figure 1: graph for questions 1, 2, 3, and 4

# MA U34605 Quiz 04 w/e 27/11/20

**Answer any 3 questions. Submit them using the `submit-work` program as pdfs, either handwritten and scanned, or typeset. They should be submitted before 1pm on Tuesday 1 December. All questions carry 20 marks.**

(1) Simulate the bcc algorithm on the graph in Figure 1, beginning at vertex 0. Show the order in which the calls to dfs begin and end, the preorder rank, and the highpt function.

(2) Apply the DFS from the Hopcroft-Tarjan algorithm to the graph in Figure 1. Begin at vertex 0. Show the order in which calls to dfs begin and end, and calculate parent, preorder rank, highpt1 and highpt2.

Then relabel the vertices, the parent function values, and the two highpt function values by the preorder ranks and tabulate them (in the order of preorder ranks).

(3) Continuing (2): Tabulate the $\phi$- function, bearing in mind that vertices are labelled by their preorder ranks.

Then construct the palm tree version of the graph, with the out-edges ordered according to the $\phi$ function.

(4) Continuing (3): Extract the paths recursively in the same order as they would be extracted in the recursive attempt-layout, and for each path say whether it is tied to its parent path (except for the first path).

(5) Give a partial proof that a plane embedded graph $G$ with $n$ vertices, assuming $n \geq 3$, has at most $3n - 6$ edges and $2n - 4$ faces. Note without proof:

- If $G$ has at least 3 edges, then every face is incident to at least 3 edges.
- Every edge is incident to 1 or 2 faces.
- Euler formula: $v - e + f = c + 1$ where $G$ has $v$ vertices, $e$ edges, $f$ faces, and $c$ components.