

MA U34605 Quiz 02 w/e 23/10/20

Answer any 3 questions. Submit them using the submit-work program as pdfs, either handwritten and scanned, or typeset. They should be submitted before 1pm on Tuesday 27 October. All questions carry 20 marks.

(1) As in the first quiz, there is some ambiguity in the word ‘sorted.’ How does this impact on the use of search trees? There are reasons to consider this rather different from the earlier question about binary search.

(2) Sorting usually allows for the same key (or item) to have repetitions in the input, and in the output. A method is *stable* if, when a key is repeated in the output, it is repeated in the same order as in the input.

Is mergesort, defined by the code presented in the notes, stable? Give reasons.

(3) (Splay trees). Let T_1 be a tree with p nodes, and T_2 a tree with q nodes, with root x , where x has no left child. Join the trees by making T_1 the left subtree of x . Calculate the change in potential.

(4) Here is one way — there might be better ways — to show that the IPL of a binary tree is $\Omega(n \log n)$. Note: n is always the number of nodes in the tree.

It is enough to assume that there exists a natural number d such that (i) all leaves are at depths d or $d + 1$, and (ii) there are 2^d nodes at level d .

Prove that if T is any binary tree in which there is a node u with fewer than 2 children, and there is a leaf v such that $\text{depth}(v) - \text{depth}(u) > 1$, then the tree can be altered, while retaining the number of nodes, to get another tree with smaller IPL.

(5) Fix the double red illustrated, assuming that the whole tree fulfils the red-black conditions everywhere else.

In your answer you should label the nodes $a \dots h$ (in inorder), and show their ranks, given that the lowest red node has rank $r - 1$.

