Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Mathematics

JF Science **Trinity Term 2019**

MAU11S02: Maths for Scientists II

Wednesday 24th April **09.30 - 13.00**

Profs. Miriam Logan and Colm Ó Dúnlaing

Instructions to Candidates:

Attempt 3 questions from Section A and 3 questions from Section B.

Show all work.

Remember to fold down and glue the flap on every answer booklet.

Non-programmable calculators are permitted for this exam.

Mathematical tables are available from the invigilators.

You may not start this examination until you are instructed to do so by the Invigilator.

SECTION A

1. (a) [**8 marks**] Calculate the following determinant, by cofactor expansion along the fourth row.

$$\begin{vmatrix} 1 & 1 & -1 & 4 \\ -1 & -2 & -1 & 0 \\ -1 & -1 & 2 & -6 \\ -1 & -4 & -7 & 10 \end{vmatrix}$$

   (b) [**6 marks**] Compute the above determinant by reducing to upper triangular form.

   (c) [**6 marks**] Compute bases for the row space, column space, and kernel (nullspace) of the following matrix

$$\begin{bmatrix} 0 & 2 & 4 & 4 & 14 \\ -1 & 1 & 0 & 3 & 7 \\ 1 & 1 & 4 & 3 & 11 \end{bmatrix}$$

2. (a) [**7 marks**] Construct a right-handed orthonormal basis for $\mathbb{R}^3$ whose third vector X_3 is in the direction $(-1, 1, 1)$

   (b) [**3 marks**] Give the matrix for 60° rotation around the z-axis in $\mathbb{R}^3$.

   (c) [**10 marks**] Hence or otherwise calculate the matrix for 60° rotation around the axis through $(-1, 1, 1)$.

3. (a) [**8 marks**] Given the following list of data points in $\mathbb{R}^2$

$$(-1, 1),\ (0, 2),\ (1, 1),\ (3, 3)$$

   calculate the linear least-squared error estimation, the line $y = mx + c$, by reducing to linear equations in m and c, and solving.

   (b) [**12 marks**] Given the same four data points, calculate the quadratic least-squared error estimation, the curve $y = ax^2 + bx + c$, by reducing to linear equations in a, b, and c, and solving.

4. (a) [**6 marks**] Two 4-sided 'dice' are thrown. They are regular tetrahedrons, red and blue, each of whose faces are labelled 1 to 4. Let i (respectively, j) be the number

on the face on which the red (respectively, blue) tetrahedron lands. Assuming i and j take the values 1 to 4 independently with uniform probability, (i) find the probability distribution p_k, $2 \leq k \leq 8$, where $k = i + j$; (ii) find its mean; and (iii) find its variance.

(b) [**4 marks**] Calculate the sample mean and sample standard deviation of the following numbers

`4.22 6.85 6.17 1.17 4.27 7.52 4.28`

(c) [**4 marks**] The above numbers are distributed $N(\mu, \sigma^2)$ where μ and σ^2 are unknown. Using Student's t-distribution, give a 95% symmetric 2-sided confidence interval for μ.

(d) [**6 marks**] Using the chi-squared distribution, give a 90% 2-sided confidence interval both for σ^2 and σ.

SECTION B

5. (a) Evaluate the following integrals:

   i. [**5 marks**]

$$\int_{-2}^{2} \sqrt{8-x^2}\, dx$$

   ii. [**5 marks**]

$$\int \frac{x}{\sqrt{1+2x}}\, dx$$

   iii. [**5 marks**]

$$\int_{0}^{\frac{1}{2}} \frac{5x+1}{2x^2-x-1}\, dx$$

   Write your answer in the form $\frac{a}{b}\ln(b)$ where a, b are integers.

   iv. [**5 marks**]

$$\int_{1}^{\infty} \frac{\ln x}{x^3}\, dx$$

6. (a) [**6 marks**] Solve the initial value problem:

$$\frac{dy}{dx} = \frac{xy^4}{\sqrt{1+x^2}} \qquad y(0) = 1$$

   (b) [**6 marks**] Solve the initial value problem:

$$\frac{dy}{dx} - \frac{3}{x}y = 2x^3 e^{2x} \qquad y(1) = 0$$

   (c) [**4 marks**] Put the following numbers in increasing order

$$L_{10}, R_{10}, M_{100}, T_{100}, \int_{1}^{3} x\, e^x\, dx$$

   (Recall: L_n is the left end point approximation, R_n is the right end point approximation, M_n is the mid point approximation and T_n is the trapezoidal approximation of the integral $\int_{1}^{3} x\, e^x\, dx$, each using n subintervals.)

   (d) [**4 marks**] Suppose C is the curve given by the parametric equations

$$x = t - \frac{1}{t} \qquad y = 1 + t^2$$

   Find the equation of the tangent line to the curve at the point $t = 1$.

7. (a) [**4 marks**] Determine whether the following series converges or diverges. If it converges find the sum of the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n (3\pi)^{2n+1}}{(2n+1)!\,(2)^{2n+1}}$$

(b) Determine whether the following series converge or diverge. Give reasons for your answer.

(i) [**5 marks**]

$$\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{2n}}{\sqrt{n+7}}$$

(ii) [**5 marks**]

$$\sum_{n=1}^{\infty} \frac{n(\sin n)^2}{n^4+5}$$

(c) [**6 marks**] Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^n}{2^n(n+1)^2}$.

8. (a) [**5 marks**] Find the Maclaurin polynomial of degree 19 for the function

$$f(x) = x\cos(x^3).$$

(b) [**5 marks**] Find the length of the curve $y = x^2 - \frac{1}{8}\ln(x)$ for $1 \le x \le e$.

(c) [**6 marks**] Find the area of the region that is bounded by $y = \sqrt{x} - 1$ and $x - y = 1$.

(d) [**4 marks**] Are the following statements true or false? If they are true explain why, if they are false, explain why or give a counterexample.

(i) If $\lim_{n\to\infty} a_n = 0$ then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.

(ii) Suppose $f(x)$ is continuous on the interval $[-2, 1]$ and $f(x) \ge 0$ for all $x \in [-2, 1]$ then $\displaystyle\int_{-2}^{1} f(x)\,dx \ge 0$.

(iii) If the n^{th} partial sum of the series $\displaystyle\sum_{n=1}^{\infty} a_n$ is given by

$$s_n = a_1 + a_2 + a_3 + \cdots + a_n = \frac{3n^2}{n^2+4}$$

then the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.

(iv) $\displaystyle\int_{-1}^{1} \sqrt{1-x^2}\,dx = \pi$.