nxn determinants There is a recusive formula

Let A = [a_ij] _nxn be a square matrix. We use double indexing in the usual way.

$n=1$ $[a_{11}]$ $n=2$ $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $n=3$ $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

if $A_{1\times 1} = [a_{11}]$ then

$$\det A = a_{11}$$

if $n > 1$, $A_{n\times n} = [a_{ij}]$ then

$$\det A = \sum_{j=1}^{n} a_{1j} (-1)^{j+1} \operatorname{minor}_{1j}(A) = \sum_{j=1}^{n} a_{1j} \operatorname{cofactor}_{1j}(A)$$

We can handle $n=2$

$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

1,1 minor a_{22} 1,1 cofactor a_{22}

1,2 minor a_{21} 1,2 cofactor $-a_{21}$

$a_{11}a_{22} - a_{12}a_{21}$ $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

same as before.

$n=3$ $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ or: $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$

minors 1,1 $\begin{vmatrix} \cancel{a} & \cancel{b} & \cancel{c} \\ \cancel{d} & e & f \\ \cancel{g} & h & i \end{vmatrix}$ 1,2 $\begin{vmatrix} \cancel{a} & \cancel{b} & \cancel{c} \\ d & \cancel{e} & f \\ g & \cancel{h} & i \end{vmatrix}$ 1,3 $\begin{vmatrix} \cancel{a} & \cancel{b} & \cancel{c} \\ d & e & \cancel{f} \\ g & h & \cancel{i} \end{vmatrix}$

$\det = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$ same as before $P\cdot(Q\times R)$

4x4 determinants are a long calculation

·a_11 x minor_11 - a_12 x minor_22 + a_13 x minor_13 - a_14 x minor_14

$$\begin{vmatrix} 1 & -2 & 6 & 3 \\ -3 & 7 & -20 & 1 \\ -1 & 3 & -7 & 4 \\ -2 & 3 & -13 & 5 \end{vmatrix}$$

1,1 minor $\begin{vmatrix} 7 & -20 & 1 \\ 3 & -7 & 4 \\ 3 & -13 & 5 \end{vmatrix} = (7,-20,1) \cdot (17,-3,-18) = 161$

$(1,2): \begin{vmatrix} -3 & -20 & 1 \\ -1 & -7 & 4 \\ -2 & -13 & 5 \end{vmatrix}$

$= (-3,-20,1) \cdot (17,-3,-1) = 8$

$(1,3): \begin{vmatrix} -3 & 7 & 1 \\ -1 & 3 & 4 \\ -2 & 3 & 5 \end{vmatrix}$

$= (-3,7,1) \cdot (3,-3,3) = -27$

$(1,4): \begin{vmatrix} -3 & 7 & -20 \\ -1 & 3 & -7 \\ -2 & 3 & -13 \end{vmatrix}$

$= (-3,7,-20) \cdot (-18,1,3) = 1$

1×161
$+ 2 \times 8$
$+ 6(-27)$
$- 3 \times 1$
$= 177 - 165$
$= 12$

determinant is 12