Adjoint of a 2x2 matrix

Suppose P,Q are the rows of a 2x2 matrix. The adjoint of the matrix is defined as

$$\begin{bmatrix} \uparrow & \uparrow \\ N_Q^T & -N_P^T \end{bmatrix}$$

$$\underbrace{N_Q^T \quad -N_P^T}$$

Transposed to column vectors.

EG:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad \operatorname{adj} A = \begin{bmatrix} -4 & 2 \\ 3 & -1 \end{bmatrix}$$

$$A \operatorname{adj} A = \begin{bmatrix} \det A & 0 \\ 0 & \det A \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A} \operatorname{adj} A$$

Adjoint and Cramer's Rule summed up

matrix $A = \begin{bmatrix} P & Q \end{bmatrix}$ (columns)

$|A|$ or det A : determinant $= -P \cdot N_Q = N_P \cdot Q$

or $|P\ Q| \ldots$

$A \begin{bmatrix} x \\ y \end{bmatrix} = R \qquad x = \dfrac{|R\ Q|}{|A|} \qquad y = \dfrac{|P\ R|}{|A|}$

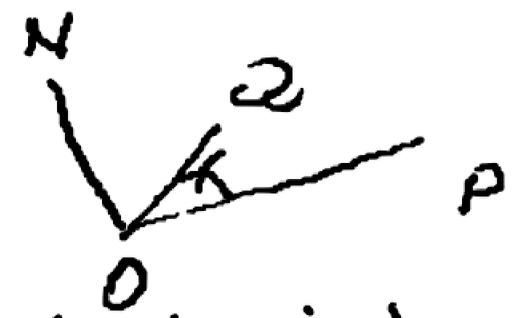
adjoint $A = \begin{bmatrix} N_Q^T \\ -N_P^T \end{bmatrix}$ (rows) $\quad A^{-1} = \dfrac{1}{|A|}$ adjoint A

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}^{-1} = \frac{1}{5} \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$$

Cross product corresponds to the positive normal in 2 dimensions. Given P, Q in $\mathbb{R}^3$, $\vec{OP} \times \vec{OQ} = \vec{ON}$

where $* \, |\vec{ON}|$ = area of parall^{m} $O, P, P+Q, Q$

(0 if O, P, Q collinear)



* If not, $\vec{ON}$ is $\perp$ this parall^{m}

* if not, P, Q, N right-handed.

(viewed from N, angle $P\hat{O}Q$ anticlockwise).

FACT (write $P \times Q$ etcetera)

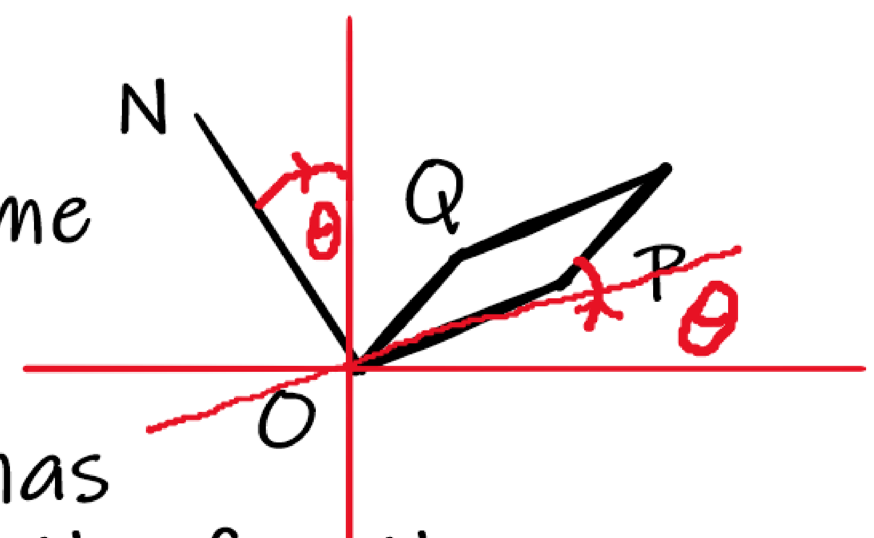
$$(a, b, c) \times (d, e, f) = $$

$$\left(\begin{vmatrix} b & c \\ e & f \end{vmatrix}, -\begin{vmatrix} a & c \\ d & f \end{vmatrix}, \begin{vmatrix} a & b \\ d & e \end{vmatrix} \right)$$

SIGN!!

It is possible to explain the formula. Given P,Q, and N = PxQ (assume nonzero), project N horizontally onto the z-axis and the parallelogram vertically onto the xy-plane.

The length of ON, and the area of the parallelogram, are both multiplied by the same factor cos theta



Say P=(a,b,c)
Q = (d,e,f)

The projected parallelogram has corners O, (a,b,0), (d,e,0), and the fourth corner. Its area is +/- (ae-bd), ... as given. The projection of N is the third component of PxQ.

Purely for interest, not for examination.