U11S02 lin alg. + stats.

Section 1 notation / outline syllabus

check 1.1 U11S01 — prerequisites

1.2 outline

1.3 Notation $\mathbb{N} = \{0, 1, 2 \dots\}$

$\boxed{\mathbb{R} \text{ (reals)}}$ $\mathbb{C}$ complex $\mathbb{Q}$ rationals

point P in $\mathbb{R}^2$ or $\mathbb{R}^3$ displacement $\overrightarrow{PQ}$ vector $\vec{v}$

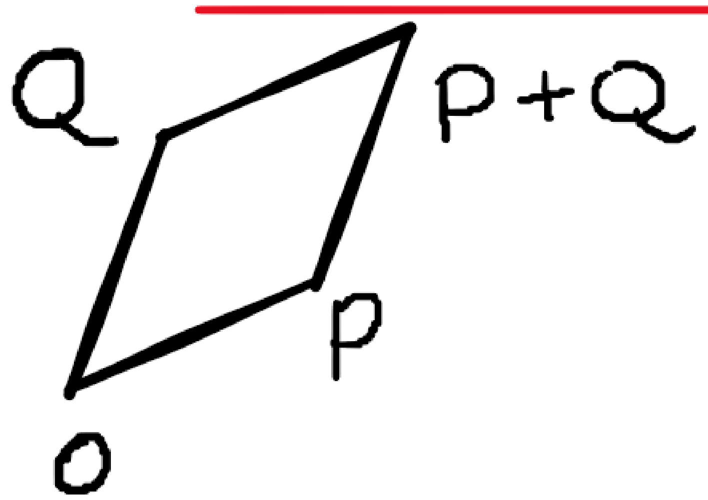
$\overrightarrow{OP} \cdot \overrightarrow{OQ}$ or $P^T Q$ (if column vectors)

or $P \cdot Q$

Triple product $\overrightarrow{OP} \cdot (\overrightarrow{OQ} \times \overrightarrow{OR})$ or $P \cdot (Q \times R)$

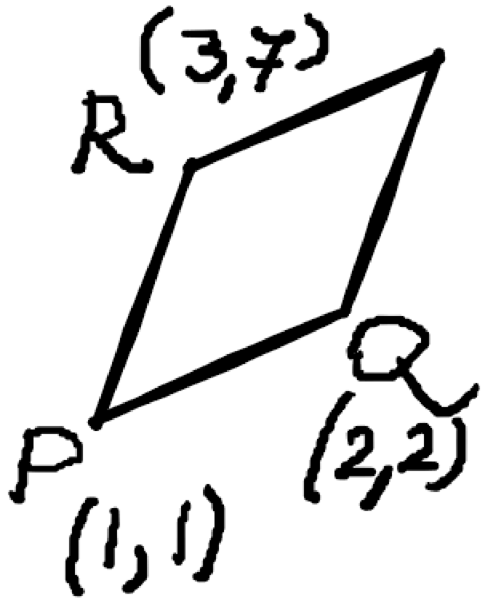
2: determinants in 2 and 3 dimensions

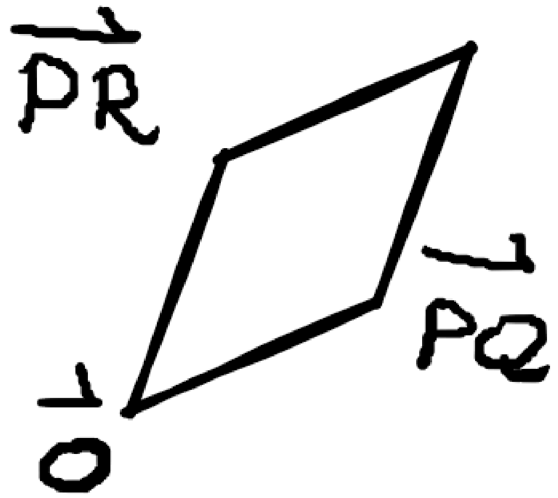
$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \qquad \begin{vmatrix} 2 & 7 \\ 1 & 8 \end{vmatrix} = 2\times 8 - 1\times 7 = 16 - 7 = 9$$

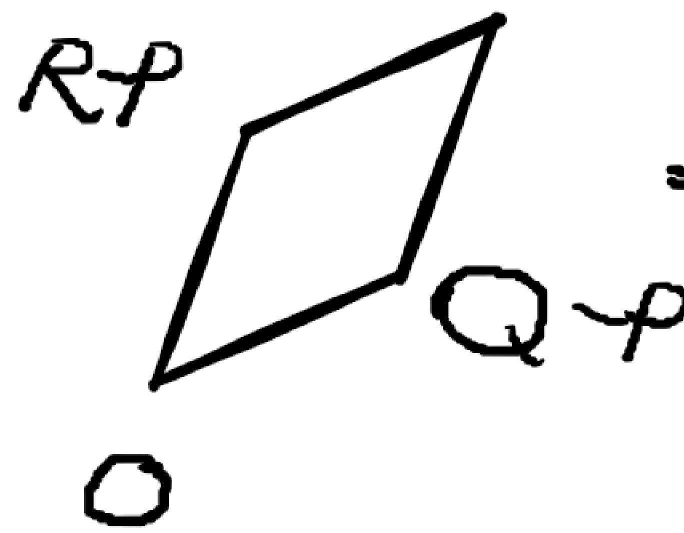


$P = (a,b) \quad Q = (c,d)$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \pm \text{ area of parallelogram.}$$





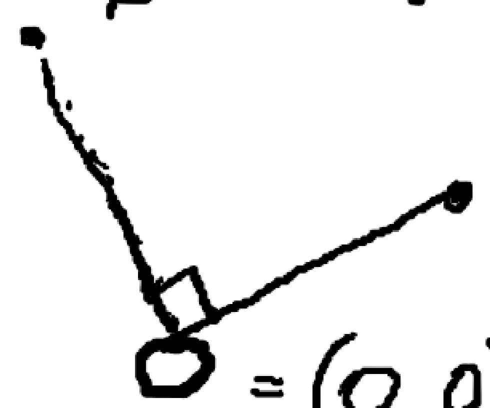


$$\Delta = \frac{1}{2} \|^{lm}$$

$$= \pm\frac{1}{2} \begin{vmatrix} 1 & 1 \\ 2 & 6 \end{vmatrix}$$

$$= \frac{1}{2}\times 4 = 2$$

$N_p = (-b, a)$



$P = (a, b)$

$O = (0, 0)$

The positive normal N_p

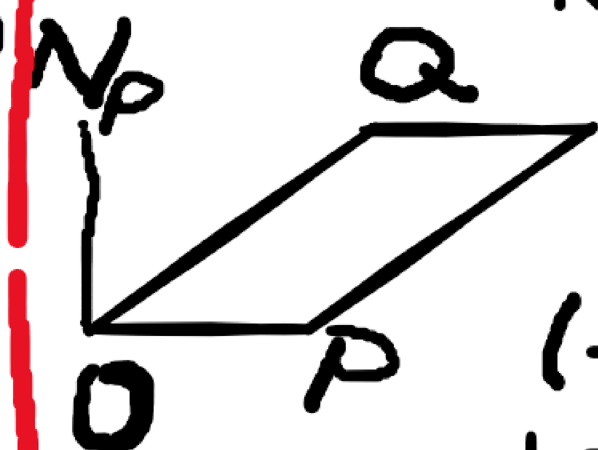
$(P \neq 0)$

- $|N_p| = |P|$
- $\overrightarrow{ON_p} \perp \overrightarrow{OP}$
- 90^0 anticlockwise.

$P = (a, b)$ $\quad Q = (c, d)$

$N_p \cdot Q =$

$- P \cdot N_Q$



$N_p \cdot Q =$

$(-b, a) \cdot (c, d)$

$= \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

Cramer's Rule

$$\begin{array}{c c} P \rightarrow & \\ Q \rightarrow & \end{array}\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{array}{c} \begin{vmatrix} a & c \\ b & d \end{vmatrix} \\ \begin{array}{cc} \uparrow & \uparrow \\ P & Q \end{array} \end{array} = \begin{array}{c} \begin{vmatrix} a & b \\ c & d \end{vmatrix} \\ \begin{array}{cc} \uparrow & \uparrow \\ P & Q \end{array} \end{array}$$

$$ax + by = c$$
$$dx + ey = f$$

$$P = \begin{bmatrix} a \\ d \end{bmatrix} \quad Q = \begin{bmatrix} b \\ e \end{bmatrix} \quad R = \begin{bmatrix} c \\ f \end{bmatrix}$$

$$Px + Qy = R$$

$$N_p \cdot Px + N_p \cdot Qy = N_p \cdot R$$

$$N_p \cdot P = 0$$

$$\rightarrow (N_p \cdot Q)\, y = N_p \cdot R$$

$$y = \frac{N_p \cdot R}{N_p \cdot Q} = \begin{vmatrix} a & c \\ d & f \end{vmatrix} \div \begin{vmatrix} a & b \\ d & e \end{vmatrix}$$

Similarly $$x = \frac{-R \cdot N_Q}{-P \cdot N_Q} = \frac{\begin{vmatrix} c & b \\ f & e \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}$$

EG $x + 3y = 2$, $x + 7y = 3$

$$x = \begin{vmatrix} 2 & 3 \\ 3 & 7 \end{vmatrix} \div \begin{vmatrix} 1 & 3 \\ 1 & 7 \end{vmatrix} = \frac{5}{4}$$

$$y = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} \div \begin{vmatrix} 1 & 3 \\ 1 & 7 \end{vmatrix} = \frac{1}{4}$$

Check

$$\frac{5}{4}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{1}{4}\begin{bmatrix} 3 \\ 7 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \checkmark$$

Adjoint of a 2x2 matrix

Suppose P,Q are the rows of a 2x2 matrix. The adjoint of the matrix is defined as

$$\cancel{N_Q} \quad \underbrace{\begin{bmatrix} N_Q^T & -N_P^T \end{bmatrix}}$$

Transposed to column vectors.

EG: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ adj $A = \begin{bmatrix} -4 & 2 \\ 3 & -1 \end{bmatrix}$

$$A \operatorname{adj} A = \begin{bmatrix} \det A & 0 \\ 0 & \det A \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A} \operatorname{adj} A$$