MAU11S02 Eighth Friday quiz, week 11
Friday 8/4/22 ANSWERS

Rules and procedures.
1. Attempt 3 questions. Only *your first three answers* will be marked. **2.** Each question carries 20 marks, so the maximum quiz mark is 60. **3.** If a particular method of solution is stipulated, you get no marks if you don't use it. **4.** ***Show all work.*** No marks will be given for answers which do not show the calculations. **5.** Your answers should be scanned and submitted to Blackboard as a 'Friday assignment.'

Question 1. $X \sim B(50, 2/3)$. Use the normal approximation, not forgetting the continuity correction, to estimate $P(15 < X \leq 25)$. Calculate the more accurate estimate using the tables in the online class notes.
Answer.

$$X \approx N(100/3, 100/9)$$
$$Z = \frac{X - 100/3}{10/3}$$
$$X \in [15.5, 25.5] \quad \text{(continuity correction)}$$
$$Z \in \left[\frac{15.5 - 100/3}{10/3}, \frac{25.5 - 100/3}{10/3}\right]$$
$$= [-5.15, -2.35]$$
$$P(-5.15) = 0, \quad P(-2.35) = 1 - .9906$$

Answer .0094. Answer from the accurate table is .0108.
Question 2. $X \sim B(50, 1/2)$. Use the normal approximation, not forgetting the continuity correction, to estimate $P(15 < X \leq 25)$. Calculate the more accurate estimate using the tables in the online class notes.
Answer.

$$X \approx N(25, 12.5)$$
$$Z = \frac{X - 25}{\sqrt{12.5}}$$
$$X \in [15.5, 25.5]$$
$$Z \in \left[\frac{15.5 - 25}{\sqrt{12.5}}, \frac{25.5 - 25}{\sqrt{12.5}}\right]$$
$$= [-2.687, 0.1414]$$
$$P(.1414) - P(-2.687) = .5557 - .0036 = .5521.$$

Answer .5521. Answer from the accurate table is .5528.
Question 3. Compute the sample average and the sample standard deviation (use the correct formula) of

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-0.08 4.82 2.79 2.19 4.64 4.95 -0.54 4.05 3.45 3.75 1.87
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Answer.

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n 11 Sample mean 2.8991 sample variance 3.5555 sdev 1.8856
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Question 4. The sample in Question 3 is from independent random variables, all $\sim N(\mu, \sigma^2)$. Neither μ nor σ is known. Use Student's t-distribution to give 90% and 95% confidence intervals for μ.

Answer.

$$\sqrt{11}\frac{2.8991 - \mu}{1.8856} \sim t_1 0$$
$$\mu - 2.8891 \in [\mp\frac{1.8856\alpha}{\sqrt{11}}]$$
$$\mu \in 2.8891 \mp .5685\alpha$$

Where $\alpha = 1.812$ for 90% confidence. This comes by symmetry from the 10-degrees-of-freedom row in the official tables; the tail from 1.812 has probability 5%.

And $\alpha = 2.228$ for 95% confidence.

For 90% confidence:

$$[1.869, 3.929]$$

and 95%:

$$[1.632, 4.166]$$

Question 5. It is believed that there are 150 fish in a certain pond. Take this as the null hypothesis, and the alternative hypothesis is that there are more than 150 fish in the pond.

To test this hypothesis, 50 fish are caught, tagged, and returned to the pond. After all have been returned, 50 are again caught and returned: 10 of them were tagged.

The test then is whether this is consistent with $B(50, 1/3)$.

(i) State the null and alternative hypotheses. (In both cases a binomial distribution is assumed.)

(ii) Does this support the alternative hypothesis at the 5% level of significance?

Answer.

(i) The distribution is $B(50, p)$; $H_0 : p = 1/3$; $H_1 : p < 1/3$.

We use the normal approximation to gauge the probability that $X \leq 10$, with the continuity correction. The correction transforms this to $X \leq 10.5$.

$$Z = \frac{X - 50/3}{10/3}$$
$$X \leq 10.5$$
$$Z \leq \frac{10.5 - 50/3}{10/3} = -1.85$$

This last has probability .0322, so we accept that there are more than 150 fish in the pond.