MAU11S02 seventh Monday quiz, week 10
Monday 28/3/22 ANSWERS

Rules and procedures.
1. Attempt 3 questions. Only *your first three answers* will be marked. **2.** Each question carries 20 marks, so the maximum quiz mark is 60. **3.** If a particular method of solution is stipulated, you get no marks if you don't use it. **4.** ***Show all work.*** No marks will be given for answers which do not show the calculations. **5.** Your answers should be scanned and submitted to Blackboard as a 'Monday assignment.'

Question 1. Given independent random variables $X_1 \sim B(3, 1/2)$ and $X_2 \sim B(3, 2/3)$, calculate the probabilities $X_1 = x_1$ and $X_2 = x_2$, $0 \leq x_1, x_1 \leq 3$.
Answer.

	0	1	2	3
0	1/216	1/36	1/18	1/27
1	1/72	1/12	1/6	1/9
2	1/72	1/12	1/6	1/9
3	1/216	1/36	1/18	1/27

Question 2. Calculate the probability distribution of the random variable $X_1 + X_2$.
Answer.

0	1	2	3	4	5	6
1/216	1/24	11/72	7/24	11/36	1/6	1/27

Question 3. Here is the graph of a continuous distribution. (i) What is b? (ii) Given $a = 2$, evaluate $E(X)$, the mean of the distribution.

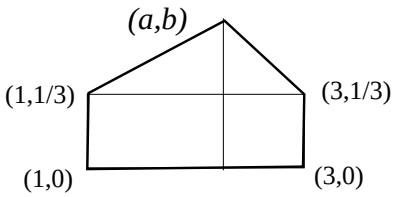

Figure 1: Question 3

Answer. (i) The area is $2/3 + (1/2)(2)(b - 1/3) = b + 1/3$, and this equals 1, so $b = 2/3$.
(ii)

$$\text{pdf}: \begin{cases} x/3 & (1 \leq x \leq 2) \\ 4/3 - x/3 & (2 \leq x \leq 3) \end{cases}$$

$$\int_1^2 \frac{x^2}{3} dx = \left[\frac{x^3}{9}\right]_1^2$$

$$\int_2^3 \left(\frac{4x}{3} - \frac{x^2}{3}\right) dx = \left[\frac{4x^2}{6}\right]_2^3 - \left[\frac{x^3}{9}\right]_2^3$$

$$\text{Answer} \quad \frac{8}{9} - \frac{1}{9} + \frac{4 \times 9}{6} - \frac{4 \times 4}{6} - \frac{27}{9} + \frac{8}{9} = 2$$

Question 4. Suppose $X \sim N(1,4)$. Evaluate the following (P means probability). (i) $P(X \leq 3)$. (ii) $P(1 < X < 3)$. (iii) $P(X \geq 2)$. (iv) $P(-3 \leq X \leq -2)$. (v) $P((-3 \leq X \leq -2)\ \&\ (X \neq -3))$.

Answers.

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Z = (X-1)/2

(i) Z<=1, .8413 (ii) 0<=Z<=1 .3413  (iii )Z>=1/2  .3085
(iv) -2 <= Z <= -1.5  ...  .9772 - .9332 = .0440
(v) .0440, exactly the same.
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Question 5. Here is a distribution with infinite discrete sample space 'parametrised' by α, where $0 < \alpha < 1$. Evaluate $E(X)$, the mean of the distribution.

$$p_i = (1-\alpha)\alpha^{i-1}, \qquad i = 1, 2, \ldots$$

Hint: evaluate $\frac{d}{d\alpha}\frac{1}{1-\alpha}$.

Answer.

$$\frac{d}{d\alpha}\frac{1}{1-\alpha} =$$
$$\frac{1}{(1-\alpha)^2} =$$
$$\frac{d}{d\alpha}(1+\alpha+\alpha^2+\ldots) =$$
$$1+2\alpha+3\alpha^2\ldots =$$
$$\sum_{i\geq 1} i \times \alpha^{i-1}$$

Multiply by $(1-\alpha)$ and you have the the expectation, i.e., $1/(1-\alpha)$.