MAU11S02 seventh Friday quiz, week 10
Friday 1/4/22 ANSWERS

Rules and procedures.
1. Attempt 3 questions. Only *your first three answers* will be marked. **2.** Each question carries 20 marks, so the maximum quiz mark is 60. **3.** If a particular method of solution is stipulated, you get no marks if you don't use it. **4.** ***Show all work.*** No marks will be given for answers which do not show the calculations. **5.** Your answers should be scanned and submitted to Blackboard as a 'Friday assignment.'

Question 1. Given independent random variables $X_1 \sim B(3, 1/2)$ and $X_2 \sim B(3, 3/5)$, calculate the probabilities $X_1 = x_1$ and $X_2 = x_2$, $0 \leq x_1, x_1 \leq 3$.
Answer.

	0	1	2	3
0	$\frac{1}{125}$	$\frac{9}{250}$	$\frac{27}{500}$	$\frac{27}{1000}$
1	$\frac{3}{125}$	$\frac{27}{250}$	$\frac{81}{500}$	$\frac{81}{1000}$
2	$\frac{3}{125}$	$\frac{27}{250}$	$\frac{81}{500}$	$\frac{81}{1000}$
3	$\frac{1}{125}$	$\frac{9}{250}$	$\frac{27}{500}$	$\frac{27}{1000}$

Question 2. Calculate the probability distribution of the random variable $X_1 + X_2$.
Answer.

0	1	2	3	4	5	6
1/125	3/50	93/500	305/1000	279/1000	135/1000	27/1000

Question 3. Here is the graph of a continuous distribution, whose pdf is formed of two quadratic curves, crossing the x-axis at -1 and 2 respectively, and with a common horizontal tangent at $(0, h)$.

(i) What is h? (ii) Calculate the mean of the distribution.
Answer. PDF p:

$$p(x) = \begin{cases} h \times (1 - x^2) & -1 \leq x \leq 0 \\ h \times (1 - x^2/4) & 0 \leq x \leq 2 \end{cases}$$

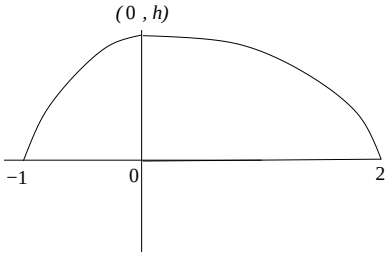

Figure 1: Question 3

(i)

$$\int_{-1}^{0} h(1-x^2)dx = [x - x^3/3]_{-1}^{0} = 2h/3$$
$$\int_{0}^{2} h(1-x^2/4)dx = [x - x^3/12]_{0}^{2} = 4h/3$$
$$\int_{-1}^{2} = 2; \quad h = 1/2$$

(ii)

$$\frac{1}{2}\int_{-1}^{0}(x-x^3)dx + \frac{1}{2}\int_{0}^{1}(x-x^3/4)dx =$$
$$\frac{1}{2}\left([x^2/2 - x^4/4]_{-1}^{0} + [x^2/2 - x^4/16]_{0}^{1}\right) =$$
$$-1/4 + 1/2 = 1/4$$

Question 4. Suppose $X \sim N(2, 9)$. Evaluate the following (P means probability). (i) $P(X \leq 3)$. (ii) $P(1 < X < 3)$. (iii) $P(X \geq 2)$. (iv) $P(-3 \leq X \leq -2)$. (v) $P((-3 \leq X \leq -2)\ \&\ (X \neq -3))$.

Answers.

$Z = (X - 2)/3$:

x	-3	-2	1	2	3
z	$-5/3$	$-4/3$	$-1/3$	0	$1/3$
$P(Z \leq z)$	.0475	.1018	.3707	.5	.6293

(i) .6293, (ii) .2586, (iii) .5, (iv) .0543, (v) .0543

Question 5. Here is a distribution with infinite discrete sample space 'parametrised' by α, where $0 < \alpha < 1$. Evaluate $E(X)$, the mean of the distribution.

$$p_i = (1-\alpha)\alpha^{i-1}, \qquad i = 1, 2, \ldots$$

Hint: evaluate $\frac{d}{d\alpha}\frac{1}{1-\alpha}$.

Answer.

$$\frac{d}{d\alpha}\frac{1}{1-\alpha} =$$
$$\frac{1}{(1-\alpha)^2} =$$
$$\frac{d}{d\alpha}(1+\alpha+\alpha^2+\ldots) =$$
$$1+2\alpha+3\alpha^2\ldots =$$
$$\sum_{i\geq 1} i \times \alpha^{i-1}$$

Multiply by $(1-\alpha)$ and you have the the expectation, i.e., $1/(1-\alpha)$.