MAU11S02 sixth Monday quiz, week 9
Monday 21/3/22 due 12 noon Thursday 24/3/22

Rules and procedures.
1. Attempt 3 questions. Only *your first three answers* will be marked. **2.** Each question carries 20 marks, so the maximum quiz mark is 60. **3.** If a particular method of solution is stipulated, you get no marks if you don't use it. **4.** ***Show all work.*** No marks will be given for answers which do not show the calculations. **5.** Your answers should be scanned and submitted to Blackboard as a 'Monday assignment.'

Question 1. Calculate the quadratic function $y = ax^2 + bx + c$ best fitting the following data (least squared error estimate). ***You must use the formula given in lectures, and show the calculations.***

$$(-3, 3)\ (-2, 0)\ (0, 3)\ (1, 1)$$

Question 2. Find eigenvalues and eigenvectors, and the change-of-basis matrix S, for the matrix A, where

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$$

Question 3. Calculate e^{At}, where A is as in Question 2.
Question 4. Solve in full the differential equations

$$\frac{dx}{dt} = x + 2y; \qquad \frac{dy}{dt} = 2x - 2y$$

with $x = 4$ and $y = -3$ at $t = 0$.
Question 5. There are 4 balls in one jar, labelled $3, 1, 4, 1$ respectively, and 3 balls in another jar, labelled $2, 3, 3$ respectively.

The random variables X_1, X_2 represent the labels x_1, x_2 on two balls chosen randomly from each jar.

(i) List the *six* possible outcomes (x_1, x_2) and their probabilities.

(ii) List the *five* possible values of $x_1 + x_2$ and their probabilities.

Question 6. The least-squared-error method can be used to find best-fitting curves of many different shapes. Find the best-fitting curve of the form $y = 2^x a + b$ for the data points in Question 1.

Show that (note: $A^T Y$ was given incorrectly as $[2.375\ 4.375]^T$)

$$A^T A = \begin{bmatrix} 5.078125 & 3.375 \\ 3.375 & 4 \end{bmatrix}, \quad \text{and } A^T Y = \begin{bmatrix} 5.375 \\ 7 \end{bmatrix} \text{ (corrected)}$$

and hence solve the problem.