MAU11S02 fifth Friday quiz, week 6
Friday 4/3/22 ANSWERS

This is due on the Monday of Reading Week so the time given matches that of the Monday quiz. But it is not due until midnight.

Rules and procedures.
1. Attempt 3 questions. Only *your first three answers* will be marked. **2.** Each question carries 20 marks, so the maximum quiz mark is 60. **3.** If a particular method of solution is stipulated, you get no marks if you don't use it. **4.** ***Show all work.*** No marks will be given for answers which do not show the calculations. **5.** Your answers should be scanned and submitted to Blackboard as a 'Friday assignment.'

Question 1. Calculate an orthonormal basis X_1, X_2, X_3, where X_3 is a multiple of $[3\ 2\ 0]^T$.
Answer

$$W_3 = (3,2,0) \quad W_1 = (3,2,0) \times (0,0,1) = (2,-3,0)$$
$$W_2 = (3,2,0) \times (2,-3,0) = (0,0,-13)$$
$$S = \begin{bmatrix} \frac{2}{\sqrt{13}} & 0 & \frac{3}{\sqrt{13}} \\ -\frac{3}{\sqrt{13}} & 0 & \frac{2}{\sqrt{13}} \\ 0 & -1 & 0 \end{bmatrix}$$

Question 2. Use the $SA'S^{-1}$ formula to calculate the matrix for projection onto the plane through O perpendicular to X_3, X_3 as in Question 1.
Answer.

$$\begin{bmatrix} \frac{2}{\sqrt{13}} & 0 & \frac{3}{\sqrt{13}} \\ -\frac{3}{\sqrt{13}} & 0 & \frac{2}{\sqrt{13}} \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{2}{\sqrt{13}} & -\frac{3}{\sqrt{13}} & 0 \\ 0 & 0 & -1 \\ \frac{3}{\sqrt{13}} & \frac{2}{\sqrt{13}} & 0 \end{bmatrix} =$$
$$\begin{bmatrix} \frac{2}{\sqrt{13}} & 0 & \frac{3}{\sqrt{13}} \\ -\frac{3}{\sqrt{13}} & 0 & \frac{2}{\sqrt{13}} \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{2}{\sqrt{13}} & -\frac{3}{\sqrt{13}} & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{4}{13} & -\frac{6}{13} & 0 \\ -\frac{6}{13} & \frac{9}{13} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 3. Calculate the matrix for rotating points through 45° around the axis OX_3, X_3 as above.
Answer.

$$\begin{bmatrix} \frac{2}{\sqrt{13}} & 0 & \frac{3}{\sqrt{13}} \\ -\frac{3}{\sqrt{13}} & 0 & \frac{2}{\sqrt{13}} \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{\sqrt{13}} & -\frac{3}{\sqrt{13}} & 0 \\ 0 & 0 & -1 \\ \frac{3}{\sqrt{13}} & \frac{2}{\sqrt{13}} & 0 \end{bmatrix} =$$
$$\begin{bmatrix} \frac{2}{\sqrt{13}} & 0 & \frac{3}{\sqrt{13}} \\ -\frac{3}{\sqrt{13}} & 0 & \frac{2}{\sqrt{13}} \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{2}{\sqrt{26}} & -\frac{3}{\sqrt{26}} & \frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{26}} & -\frac{3}{\sqrt{26}} & -\frac{1}{\sqrt{2}} \\ \frac{3}{13} & \frac{2}{13} & 0 \end{bmatrix} = \begin{bmatrix} \frac{4}{13\sqrt{2}} + \frac{9}{13} & -\frac{6}{13\sqrt{2}} + \frac{6}{13} & \frac{2}{26} \\ -\frac{6}{13\sqrt{2}} + \frac{6}{13} & \frac{9}{13\sqrt{2}} + \frac{4}{13} & -\frac{3}{26} \\ -\frac{2}{\sqrt{26}} & \frac{3}{\sqrt{26}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Question 4. Calculate the matrix for rotating points through 90° around the axis OX_3, X_3 as above.

Answer.

$$\begin{bmatrix} \frac{2}{\sqrt{13}} & 0 & \frac{3}{\sqrt{13}} \\ -\frac{3}{\sqrt{13}} & 0 & \frac{2}{\sqrt{13}} \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{\sqrt{13}} & -\frac{3}{\sqrt{13}} & 0 \\ 0 & 0 & -1 \\ \frac{3}{\sqrt{13}} & \frac{2}{\sqrt{13}} & 0 \end{bmatrix} =$$

$$\begin{bmatrix} \frac{2}{\sqrt{13}} & 0 & \frac{3}{\sqrt{13}} \\ -\frac{3}{\sqrt{13}} & 0 & \frac{2}{\sqrt{13}} \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ \frac{2}{\sqrt{13}} & -\frac{3}{\sqrt{13}} & 0 \\ \frac{3}{\sqrt{13}} & \frac{2}{\sqrt{13}} & 0 \end{bmatrix} = \begin{bmatrix} \frac{9}{13} & \frac{6}{13} & \frac{2}{\sqrt{13}} \\ \frac{6}{13} & \frac{4}{13} & -\frac{3}{\sqrt{13}} \\ -\frac{2}{\sqrt{13}} & \frac{3}{\sqrt{13}} & 0 \end{bmatrix}$$

Question 5. Calculate the linear function best fitting the following data (least squared error estimate). ***You must use the formula given in lectures, and show the calculations.***

$$(-3, 1)\ (-2, 0)\ (-1, 1)\ (1, 3)$$

Answer.

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(-3, 1)   (-2, 0)   (-1, 1)   (1, 3)
A^T A, A^T Y are
  15   -5,   -1
  -5    4,    5

By elementary methods

  5  m  -4 c  = -5
  15 m  -5 c  = -1
  15 m -12 c  = -15
         7 c  = 14, c = 2
  5 m -8 = -5, m = 3/5

m = 3/5, c = 2 ... the best-fit line is
y = 3x/5 + 2.
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