MAU11S02 fourth Monday quiz, week 5
Monday 21/2/22 ANSWERS

Rules and procedures.
1. Attempt 3 questions. Only *your first three answers* will be marked. **2.** Each question carries 20 marks, so the maximum quiz mark is 60. **3.** If a particular method of solution is stipulated, you get no marks if you don't use it. **4.** ***Show all work.*** No marks will be given for answers which do not show the calculations. **5.** Your answers should be scanned and submitted to Blackboard as a 'Monday assignment.'

Question 1. Find bases for the row space and column space of the following matrix.

$$A = \begin{bmatrix} -1 & -1 & -5 & 5 & 7 \\ -3 & -5 & -21 & 19 & 25 \\ 3 & 4 & 18 & -16 & -21 \\ 1 & -1 & -1 & -3 & -7 \end{bmatrix}$$

Answer.

```
 -1  -1  -5   5   7 matrix
 -3  -5 -21  19  25
  3   4  18 -16 -21
  1  -1  -1  -3  -7

  1   0   2   0   1
  0   1   3   0   2
  0   0   0   1   2
  0   0   0   0   0  in rref
leading columns 1,2,4;  columns 1,2,4 of A basis for columns space
Nonzero rows of rref 1,2,3; basis for row space.
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Question 2. Find a basis for the nullspace (kernel) of the matrix in Question 1.
Answer.The non-leading positions are 3 and 5.

$$x_3 = s; \quad x_5 = t$$
$$x_1 + 2s + t = 0: \quad x_1 = -2s - t$$
$$x_2 + 3s + 2t = 0: \quad x_2 = -3s - 2t$$
$$x_4 + 2t = 0: \quad x_4 = -2t$$

$$\begin{bmatrix} -2s - t \\ -3s - 2t \\ s \\ -2t \\ t \end{bmatrix} = sX + tY \quad \text{where} \quad X = \begin{bmatrix} -2 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \text{and} \quad Y = \begin{bmatrix} -1 \\ -2 \\ 0 \\ -2 \\ 1 \end{bmatrix}$$

Question 3. (i) Calculate the projection of the point $(2,1,4)$ onto the line OW where $W = (1,2,2)$ (not a unit vector).

Answer.Let $V = (1/3, 2/3, 2/3)$, a unit vector.

$$(2,1,4) \mapsto (2+2+8)/3(1,2,2)/3 = 12/9(1,2,2) = 4/3(1,2,2)$$

(ii) Calculate the projection of the point $(2,1,4)$ onto the plane through O perpendicular to OW.

Answer.

$$(2,1,4) \mapsto (2,1,4) - (4/3)(1,2,2) = (2/3, -5/3, 4/3)$$

Question 4. Calculate the point obtained by rotating the point $(2,1,4)$ through $45°$ around the line OW, where $W = (1,2,2)$.

Answer.

$$Y = (4/3, 8/3, 8/3), \quad Z = (2/3, -5/3, 4/3)$$
$$W = V \times X = (1/3)(1,2,2) \times (2,1,4) = (1/3)(6,0,-3) = (2,0,-1)$$
$$Y + \frac{1}{\sqrt{2}}Z + \frac{1}{\sqrt{2}}W = (4/3,8/3,8/3) + \frac{1}{\sqrt{2}}(2/3,-5/3,4/3) + \frac{1}{\sqrt{2}}(2,0,-1) =$$
$$\left(\frac{4}{3} + \frac{2}{3\sqrt{2}}, \frac{8}{3} - \frac{5}{3\sqrt{2}}, \frac{8}{3} + \frac{4}{3\sqrt{2}}\right).$$

Question 5. Calculate the point obtained by rotating the point $(1,2,3)$ through $60°$ around the line OW, where $W = (1,-1,2)$.

Answer.

$$X = (1,2,3)$$
$$V = \frac{1}{\sqrt{6}}(1,-1,2)$$
$$Y = \frac{5}{6}(1,-1,2)$$
$$Z = (1,2,3) - (5/6, -5/6, 10/6) = (1/6, 17/6, 8/6)$$
$$W = \frac{1}{\sqrt{6}}(1,-1,2) \times (1,2,3) = \frac{1}{\sqrt{6}}(-7,-1,3)$$
$$\cos 60° = 1/2; \sin 60° = \sqrt{3}/2$$
$$Y + Z\cos 60° + W \sin 60° =$$
$$\left(\frac{5}{6}, -\frac{5}{6}, \frac{10}{6}\right) + \left(\frac{1}{12}, \frac{17}{12}, \frac{8}{12}\right) + \left(\frac{-7}{2\sqrt{2}}, \frac{-1}{2\sqrt{2}}, \frac{3}{2\sqrt{2}}\right)$$

CORRECTION. In a previous draft of the answers, the final answer was wrong. The correct answer is

$$\left(\frac{11}{12} - \frac{7}{2\sqrt{2}}, \frac{7}{12} - \frac{1}{2\sqrt{2}}, \frac{7}{3} + \frac{3}{2\sqrt{2}}\right).$$