

MAU11S02 fourth Friday quiz, week 5

Friday 25/2/22 ANSWERS

Rules and procedures.

1. Attempt 3 questions. Only *your first three answers* will be marked. 2. Each question carries 20 marks, so the maximum quiz mark is 60. 3. If a particular method of solution is stipulated, you get no marks if you don't use it. 4. **Show all work.** No marks will be given for answers which do not show the calculations. 5. Your answers should be scanned and submitted to Blackboard as a 'Friday assignment.'

Question 1. Find bases for the row space and column space of the following matrix.

$$\begin{bmatrix} 0 & 1 & 1 & 3 & 9 \\ 1 & 3 & 4 & 12 & 35 \\ 3 & 6 & 9 & 28 & 80 \\ -3 & -6 & -9 & -27 & -78 \end{bmatrix}$$

Answer.

Reduced Row-Echelon Form:

$$\begin{array}{ccccc} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

Column space: columns 1,2,4 of original matrix

Row space: rows 1,2,3 of RREF

Question 2. Find a basis for the nullspace (kernel) of the matrix in Question 1.

Answer.

$$\begin{array}{ccccc} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{array}$$

non-leading variables x_3 and x_5 Set $x_3 = s$, $x_5 = t$

$$x_1 + s + 2t = 0 \quad x_2 + s + 3t = 0 \quad x_3 = s$$

$$x_4 + 2t = 0 \quad x_5 = t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = sX + tY, \text{ where } X = \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \text{ and } Y = \begin{bmatrix} -2 \\ -3 \\ 0 \\ -2 \\ 1 \end{bmatrix},$$

Question 3.

(i) Calculate the projection of the point $(2, 1, 1)$ onto the line OW where $W = (2, -3, 6)$ (not a unit vector).

Answer.

$$V = (2/7, -3/7, 6/7), \quad (W \cdot V)V = 7/49(2, -3, 6)$$

(ii) Calculate the projection of the point $(2, 1, 1)$ onto the plane through O perpendicular to OW .

Answer.

$$(2, 1, 1) - (2/7, -3/7, 6/7) = (12/7, 10/7, 1/7)$$

Question 4. Calculate the point obtained by rotating the point $(2, 1, 1)$ through 45° around the line OW , where $W = (2, -3, 6)$.

$$Y = (2/7, -3/7, 6/7), \quad Z = (12/7, 10/7, 1/7)$$

$$W = (2/7, -3/7, 6/7) \times (2, 1, 1) = (-9/7, 10/7, 8/7)$$

$$Y + Z/\sqrt{2} + W/\sqrt{2} =$$

$$(2/7, -3/7, 6/7) + (12/(7\sqrt{2}), 10/(7\sqrt{2}), 1/(7\sqrt{2})) + (-9/(7\sqrt{2}), 10/(7\sqrt{2}), 8/(7\sqrt{2})) =$$

$$(2/7 + 3/(7\sqrt{2}), -3/7 + 20/(7\sqrt{2}), 6/7 + 9/(7\sqrt{2}))$$

Answer.

Question 5. Calculate the point obtained by rotating the point $(2, 1, 1)$ through 60° around the line OW , where $W = (2, -3, 6)$.

Answer.

$$Y = (2/7, -3/7, 6/7), \quad Z = (12/7, 10/7, 1/7) \quad W = (-9/7, 10/7, 8/7)$$

$$Y + Z/2 + W\sqrt{3}/2 =$$

$$(2/7, -3/7, 6/7) + (6/7, 5/7, 1/14) + \sqrt{3}(-9/14, 5/7, 4/7) =$$

$$(8/7 - 9\sqrt{3}/14, 2/7 + 5\sqrt{3}/7, 13/14 + 4\sqrt{3}/7)$$