MAU11S02 third Friday quiz, week 4
Friday 18/2/22 ANSWERS

Rules and procedures.
1. Attempt 3 questions. Only *your first three answers* will be marked. **2.** Each question carries 20 marks, so the maximum quiz mark is 60. **3.** If a particular method of solution is stipulated, you get no marks if you don't use it. **4.** ***Show all work.*** No marks will be given for answers which do not show the calculations. **5.** Your answers should be scanned and submitted to Blackboard as a 'Friday assignment.'

Question 1. Calculate the following determinant by cofactor expansion along the second column.

$$\begin{vmatrix} -1 & 2 & 2 & -2 \\ 2 & -5 & -4 & 6 \\ 3 & -9 & -6 & 11 \\ 1 & -1 & 0 & 2 \end{vmatrix}$$

Answer.

```
       -1          2          2         -2
        2         -5         -4          6
        3         -9         -6         11
        1         -1          0          2
1 2 minor -8 cofactor 8
2 2 minor 10 cofactor 10
3 2 minor 4 cofactor -4
4 2 minor 0 cofactor 0
determinant is 2
```

Question 2. Calculate the same determinant again, by bringing the matrix to upper triangular form.

Answer.

```
     -1          2          2          -2
      2         -5         -4           6
      3         -9         -6          11
      1         -1          0           2

     -1          2          2          -2
      0         -1          0           2
      0         -3          0           5
      0          1          2           0
```

```
 -1          2          2         -2
  0         -1          0          2
  0          0          0         -1     swap
  0          0          2          2     swap

 -1          2          2         -2
  0         -1          0          2
  0          0          2          2
  0          0          0         -1

 -1          2          2         -2
  0         -1          0          2
  0          0          2          2
  0          0          0         -1

 -1          2          2         -2
  0         -1          0          2
  0          0          2          2
  0          0          0         -1
```

```
UTF determinant -2; 1 swap; det  = 2.
```

Question 3. Calculate a basis for the plane $2x + 7y + z = 0$. That is, find column vectors S and T such that the general solution to $2x + 7y + z = 0$ is $[x, y, z]^T = sS + tT$, $s, t \in \mathbb{R}$.

Answer. $x = (-7/2)y - z$. Let $y = s, z = t$.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (-7/2)s - t \\ s \\ t \end{bmatrix} =$$

$$sS + tT \quad \text{where}$$

$$= S = \begin{bmatrix} -7/2 \\ 1 \\ 0 \end{bmatrix}, \quad \text{and} \quad T = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.$$

Question 4. The vectors $P = (1, 2, -2)$, $Q = (1, 2, -4)$, and $R = (0, -2, 2)$, are a basis for $\mathbb{R}^3$.

Find the unique α, β, γ such that $(3, 0, -2) = \alpha P + \beta Q + \gamma R$.

Answer.

```
  1  1  0  3
  2  2 -2  0
 -2 -4  2 -2  GJE...

  1  0  0  2
  0  1  0  1
  0  0  1  3  in rref
```

$\alpha = 2, \beta = 1, \gamma = 3.$

Question 5. Let A be a square matrix such that $\det(A) = 5$. (i) What is $\det(A^{-1})$? (ii) If A is a 2×2 matrix, with determinant 5, what is $\det(\mathrm{Adj}(A))$?

Answer. (i) $1/5$ ($\det A \det(A^{-1}) = 1$).

In general, for any 2×2 matrix M, and real number α,

$$\det(\alpha M) = \alpha^2 \det(M)$$

Take $\alpha = \det A$ and $M = A^{-1}$,

$$\det((\det A)A^{-1}) = (\det A)^2 \det(A^{-1}) = \det A.$$

So: $\det \mathrm{Adj}(A) = \det A = 5$.