MAU11S02 sixth Monday quiz ANSWERS
Monday 22/3/21 due 4pm Monday 29/3/21

Rules and procedures.
1. Attempt 3 questions. Only *your first three answers* will be marked. **2.** Each question carries 20 marks, so the maximum quiz mark is 60. **3.** If a particular method of solution is stipulated, you get no marks if you don't use it. **4.** ***Show all work.*** No marks will be given for answers which do not show the calculations. **5.** Your answers should be scanned and submitted to Blackboard as a 'Monday assignment.'

Remember, you must show all work.

Question 1. Calculate the least-squares linear estimate $y = mx + c$ for the data

$$(-3, 2),\ (-2, 1),\ (-1, 1),\ (1, 1)$$

Answer.

$$X = \begin{bmatrix} -3 \\ -2 \\ -1 \\ 1 \end{bmatrix}, \quad Y = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad A = \begin{bmatrix} -3 & 2 \\ -2 & 1 \\ -1 & 1 \\ 1 & 1 \end{bmatrix}, \quad A^T A = \begin{bmatrix} 15 & -5 \\ -5 & 4 \end{bmatrix}, \quad A^T Y = \begin{bmatrix} -8 \\ 5 \end{bmatrix}$$

linear $y = -x/5 + 1$

Question 2. Calculate the least-squares quadratic estimate $y = ax^2 + bx + c$ for

$$(-3, 2),\ (-2, 1),\ (-1, 1),\ (1, 1)$$

same data as in Question 1.
Answer.

$$P = \begin{bmatrix} 9 \\ 4 \\ 1 \\ 1 \end{bmatrix}, \quad Q = \begin{bmatrix} -3 \\ -2 \\ -1 \\ 1 \end{bmatrix}, \quad R = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad Y = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix},$$

$$A^T = \begin{bmatrix} 9 & 4 & 1 & 1 \\ -3 & -2 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} A^T A = \begin{bmatrix} 99 & -35 & 15 \\ -35 & 15 & -5 \\ 15 & -5 & 4 \end{bmatrix}, A^T Y = \begin{bmatrix} 24 \\ -8 \\ 5 \end{bmatrix}$$

quadratic $\quad y = 7x^2/44 + 21x/220 + 17/22$

Question 3. Let

$$A = \begin{bmatrix} -5 & 2 \\ -28 & 10 \end{bmatrix}$$

Calculate eigenvalues and eigenvectors for A.

Answer.

$$\begin{vmatrix} \lambda+5 & -2 \\ 28 & \lambda-10 \end{vmatrix} = (\lambda+5)(\lambda-10)+56 = (\lambda-2)(\lambda-3) = 0$$

$$\lambda = 2, 3$$

$$\lambda = 2, 2I - A = \begin{bmatrix} 7 & -2 \\ 28 & -8 \end{bmatrix} \quad X_1 = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$$

$$\lambda = 3, 3I - A = \begin{bmatrix} 8 & -2 \\ 28 & -7 \end{bmatrix} \quad X_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

X_1 and X_2 are eigenvectors with eigenvalues 2 and 3 respectively.

Question 4. Hence express A in the form $SA'S^{-1}$ where A' is a diagonal matrix, and evaluate e^A.

Answer.

$$A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

$$e^A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} \begin{bmatrix} e^2 & 0 \\ 0 & e^3 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} =$$

$$\begin{bmatrix} 2e^2 & e^3 \\ 7e^2 & 4e^3 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} =$$

$$\begin{bmatrix} 8e^2 - 7e^3 & -2e^2 + 2e^3 \\ 28e^2 - 28e^3 & -7e^2 + 8e^3 \end{bmatrix}$$

There are two ways of checking this answer. First, if one pretends that $e = 1$, one should get the identity matrix. Second, $\det(e^A) = e^{\mathrm{tr}A}$ (the trace). Working out $\det(e^A)$ is not worth the effort, but one can check this on a calculator. In this example, $\det(e^A) = e^5$. Numerically, we get agreement: 148.413150.

Question 5. With S, A, A' as above, evaluate $\cos(A\pi/6)$. (Recall $\cos x = 1 - x^2/2! + x^4/4! \ldots$ and refer to the connection between e^A and $e^{A'}$ in the notes.)

Answer.

$$\cos(A\pi/6) = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} \begin{bmatrix} \cos(2\pi/6) & 0 \\ 0 & \cos(3\pi/6) \end{bmatrix} \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} =$$

$$\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} \begin{bmatrix} 1/2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} =$$

$$\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1/2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 14 & -7/2 \end{bmatrix}$$