MAU11S02 fifth Friday quiz ANSWERS
Friday 12/3/21 due 1pm Friday 26/3/21

Rules and procedures.
1. Attempt 3 questions. Only *your first three answers* will be marked. **2.** Each question carries 20 marks, so the maximum quiz mark is 60. **3.** If a particular method of solution is stipulated, you get no marks if you don't use it. **4.** ***Show all work.*** No marks will be given for answers which do not show the calculations. **5.** Your answers should be scanned and submitted to Blackboard as a 'Monday assignment.'

Question 1. Calculate the result of rotating the point $(1, 0, 0)$ through the angle $120°$ around the axis through $(1, -1, 1)$.
Answer.

$$\begin{gathered} Z \quad (1/3, -1/3, 1/3) \\ Y \quad (2/3, 1/3, -1/3) \\ W \quad \left(0, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right) \\ \cos\phi - 1/2 \quad \sin\phi\sqrt{3}/2 \\ -1/3, \quad -1/6, \quad 1/6 \\ 0, \quad 1/2, \quad 1/2 \\ 1/3, \quad -1/3, \quad 1/3 \quad \text{total:} \\ (0, 0, 1) \end{gathered}$$

Question 2. Calculate the matrix rotating points through the angle $45°$ around the axis through $(0, 5, 12)$.
Answer. The change-of-basis matrix S is calculated in the usual way.

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{12}{13} & \frac{5}{13} \\ 0 & -\frac{5}{13} & \frac{12}{13} \end{bmatrix}, \quad \text{and} \quad A' = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$SA' = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{12}{13\sqrt{2}} & \frac{12}{13\sqrt{2}} & \frac{5}{13} \\ -\frac{5}{13\sqrt{2}} & -\frac{5}{13\sqrt{2}} & \frac{12}{13} \end{bmatrix} \text{ and } SA'S^{-1} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{12}{13\sqrt{2}} & \frac{5}{13\sqrt{2}} \\ \frac{12}{13\sqrt{2}} & \frac{144}{169\sqrt{2}} + \frac{25}{169} & -\frac{60}{169\sqrt{2}} + \frac{60}{169} \\ -\frac{5}{13\sqrt{2}} & -\frac{60}{169\sqrt{2}} + \frac{60}{169} & \frac{25}{169\sqrt{2}} + \frac{144}{169} \end{bmatrix}$$

Question 3. Calculate the matrix for the perpendicular projection of points onto the plane $5y + 12z = 0$.

Answer. This time we use the formula. Let $V = (0, 5/13, 12/13)$. Relating to projection onto the line OV, $X \mapsto X - (X \cdot V)V$.

$$
\begin{aligned}
(1,0,0) &\mapsto (1,0,0) \\
(0,1,0) &\mapsto (0, \frac{144}{169}, -\frac{60}{169}) \\
(0,0,1) &\mapsto (0, -\frac{60}{169}, \frac{25}{169}) \\
\text{matrix} &\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{144}{169} & -\frac{60}{169} \\ 0 & -\frac{60}{169} & \frac{25}{169} \end{bmatrix}
\end{aligned}
$$

Question 4. Calculate the result of rotating the point $(1, 2, 1)$ through $45°$ around the axis OU where $U = (2, 3, 6)$.
Answer. Normalise $V = (2/7, 3/7, 6/7)$.

$$
\begin{gathered}
Z = (4/7, 6/7, 12/7) \\
Y = (3/7, 8/7, -5/7) \\
W = (-9/7, 4/7, 1/7) \\
(Y + W)/\sqrt{2} + z \\
\left(\frac{4}{7} - \frac{6}{7\sqrt{2}}, \frac{6}{7} + \frac{12}{7\sqrt{2}}, \frac{12}{7} - \frac{4}{7\sqrt{2}}\right)
\end{gathered}
$$

Question 5. Suppose $A = SA'S^{-1}$ in the usual context, and A' is invertible (and so is A). (i) How are $\det(A)$ and $\det(A')$ related? (ii) True or false: $A^{-1} = S(A')^{-1}S^{-1}$. Give reasons for your answers.

(i) $\det(A) = \det(SA'S^{-1}) = \det S \det(A') \det(S^{-1}) = det A' \det(SS^{-1}) = \det(A')$.

(ii) Multiply out: $A(S(A')^{-1}S^{-1}) = SA'S^{-1}S(A')^{-1}S^{-1} = SA'(A')^{-1} = SIS^{-1} = I$, So the answer is yes.