MAU11S02 fifth Monday quiz ANSWERS
Monday 8/3/21 due 4pm Monday 22/3/21

Rules and procedures.
1. Attempt 3 questions. Only *your first three answers* will be marked. **2.** Each question carries 20 marks, so the maximum quiz mark is 60. **3.** If a particular method of solution is stipulated, you get no marks if you don't use it. **4.** ***Show all work.*** No marks will be given for answers which do not show the calculations. **5.** Your answers should be scanned and submitted to Blackboard as a 'Monday assignment.'

Question 1. Calculate the result of rotating the point $(1, 0, 0)$ through the angle $120°$ around the axis through $(1, 1, 1)$.
Answer.

$$Z \quad (1/3, 1/3, 1/3)$$
$$Y \quad (2/3, -1/3, -1/3)$$
$$W \quad \left(0, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$$
$$\cos\phi - 1/2 \quad \sin\phi\sqrt{3}/2$$
$$-1/3, \quad 1/6, \quad 1/6$$
$$0, \quad 1/2, \quad -1/2$$
$$1/3, \quad 1/3, \quad 1/3 \quad \text{total:}$$
$$(0, 1, 0)$$

Question 2. Calculate the matrix rotating points through the angle $60°$ around the axis through $(0, 3, 4)$.
Answer. Change of basis matrix, and A',

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4/5 & 3/5 \\ 0 & -3/5 & 4/5 \end{bmatrix}, \quad A' = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$SA' = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{4\sqrt{3}}{10} & \frac{2}{5} & \frac{3}{5} \\ -\frac{3\sqrt{3}}{10} & -\frac{3}{10} & \frac{4}{5} \end{bmatrix}, \quad A = \begin{bmatrix} \frac{1}{2} & -\frac{2\sqrt{3}}{5} & \frac{3\sqrt{3}}{10} \\ \frac{4\sqrt{3}}{10} & -\frac{1}{25} & \frac{6}{25} \\ -\frac{3\sqrt{3}}{10} & -\frac{36}{50} & \frac{41}{50} \end{bmatrix}$$

Question 3. Calculate the matrix for the perpendicular projection of points onto the plane $3y + 4z = 0$.

Answer. Change of basis matrix, and A',

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4/5 & 3/5 \\ 0 & -3/5 & 4/5 \end{bmatrix}, \quad A' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$SA' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4/5 & 0 \\ 0 & -3/5 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{16}{25} & -\frac{12}{25} \\ 0 & -\frac{12}{25} & \frac{9}{25} \end{bmatrix}$$

Question 4. Calculate the result of rotating the point $(1, 2, 1)$ through $60°$ around the axis OW_3 where $W_3 = (2, 3, 6)$.
Answer.

$$Z = (1, 2, 1).(2, 3, 6)/49(2, 3, 6) = (4/7, 6/7, 12/7)$$
$$Y = X - Z = (3/7, 8/7, -5/7)$$
$$W = (2/7, 3/7, 6/7) \times (3/7, 8/7, -5/7) = (-9/7, 4/7, 1/7)$$
$$\text{answer } (3/14, 8/14, -5/14) + ((-9)\sqrt{3}/14, 2\sqrt{3}/7, \sqrt{3}/14) + (4/7, 6/7, 12/7)$$

Question 5. (i) A' is the matrix for rotating points through angle ϕ around the positive z-axis. Evaluate $\det(A')$. (ii) Explain why every rotation matrix has the same determinant.

Answer. (i)

$$\begin{vmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{vmatrix} = (\cos^2\phi + \sin^2\phi) \times 1 = 1.$$

(ii) Every rotation matrix A is of the form $SA'S^{-1}$ where A' is as in (i). So

$$\det(A) = \det(S)(1)\det(S^{-1}) = \det(SS^{-1}) = 1.$$