MAU11S02 fourth Monday quiz ANSWERS
Monday 1/3/21 due 4pm Monday 8/3/21

Rules and procedures.

1. Attempt 3 questions. Only *your first three answers* will be marked. **2.** Each question carries 20 marks, so the maximum quiz mark is 60. **3.** If a particular method of solution is stipulated, you get no marks if you don't use it. **4.** ***Show all work.*** No marks will be given for answers which do not show the calculations. **5.** Your answers should be scanned and submitted to Blackboard as a 'Monday assignment.'

Question 1. Let

$$P = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}, \ Q = \begin{bmatrix} 3 \\ -6 \\ -8 \end{bmatrix}, \ R = \begin{bmatrix} -5 \\ 8 \\ 14 \end{bmatrix}, \ T = \begin{bmatrix} -3 \\ 3 \\ 10 \end{bmatrix}$$

Given that P, Q, R is a 'new' basis for a 'new' coordinate system, (i) Give the change-of-basis matrix S. (ii) Supposing that T gives the 'new' coordinates for a point, calculate its 'old' or 'standard' coordinates.

Answer.

```
New coordinates are ST.
   S:                  T:
  1  3 -5     times    -3       =     -44
 -2 -6  8               3              68
 -2 -8 14              10             122
```

Question 2. Continuing question 1, supposing that T gives the standard coordinates for a point, calculate its 'new' coordinates.

Answer.

```
Solve the augmented matrix
  1  3 -5 -3
 -2 -6  8  3
 -2 -8 14 10
Gauss-Jordan elimination.......  reduced row-echelon form

   1   0   0 3/2
   0   1   0   1
   0   0   1 3/2  in rref
New coordinates (3/2, 1, 3/2)
```

Question 3. Let A be the following matrix

$$\begin{bmatrix} -1 & 3 & -3 & -4 & 0 & -6 \\ 1 & -2 & 2 & 3 & 1 & 3 \\ 1 & 0 & -2 & -5 & 1 & -7 \\ 1 & -3 & 1 & -2 & -2 & 3 \\ 3 & -7 & 7 & 10 & 2 & 13 \end{bmatrix}$$

Bring to RREF *and hence* calculate bases for (i) the row and (ii) column space of A.

Question 4. Use the RREF for the matrix A of question 3 to compute a basis for the kernel (nullspace).

Answer.

```
Gauss-Jordan Elimination:
 -1  3 -3 -4  0 -6
  1 -2  2  3  1  3
  1  0 -2 -5  1 -7
  1 -3  1 -2 -2  3
  3 -7  7 10  2 13
Reduced Row-Echelon Form:
 1 0 0 1 3 0    rowspace basis
 0 1 0 2 2 0    rowspace basis
 0 0 1 3 1 0    rowspace basis
 0 0 0 0 0 1    rowspace basis
 0 0 0 0 0 0

Column space: columns 1,2,3, and 6, of A.
Kernel: needs checking
Let x_4 = s, x_5 = t.
x_1 + s + 3t = 0
x_2 = 2s + 2t = 0
x_3 + 3s + t = 0
x_4 = s
x_5 = t
x_6 = 0


Basis: [-1 -2 -3 1 0 0]^T,     [-3 -2 -1 0 1 0]^T
```

Question 5. Let $W_3 = (2, 6, 9)$. (i) Calculate a right-handed orthonormal basis X_1, X_2, X_3 where $X_3 = W_3/|W_3|$. (ii) Show the change-of-basis matrix S.

Answer.

Needs checking.

```
    W3   2  6  9       2  6  9
               1       6 -2  0
         -------       -------
         6 -2  0      18 54 -40
```

X3 = (2,6,9)/11

X1 = (3/ sqrt 10, -1/sqrt 10, 0)

X_2 = (9,27,-20)/(11 sqrt 10).

$$S = \begin{bmatrix} \frac{3}{\sqrt{10}} & \frac{9}{11\sqrt{10}} & \frac{2}{11} \\ -\frac{1}{\sqrt{10}} & \frac{27}{11\sqrt{10}} & \frac{6}{11} \\ 0 & -\frac{20}{11\sqrt{10}} & \frac{9}{11} \end{bmatrix}$$